

**SOME ASPECTS OF SHEAR AND NATURAL CONVECTIVE FLOWS
OBEYING ARRHENIUS LAW OF VISCOSITY**

by

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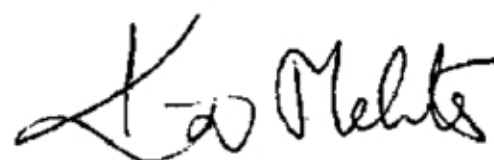
TO

MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled 'Some Aspects of Shear and Natural Convective Flows Obeying Arrhenius Law of Viscosity being submitted by Mr. Robert John D'Souza to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bona fide research work carried out by him under my supervision for the last three years and to the best of my knowledge it has reached the standard fulfilling the requirements of the regulations relating to the degree.

I further certify that the results contained in this thesis have not been submitted, either in part or in full, to any other University or Institute for the award of any Degree or Diploma.



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^a Next in the list comes my wife whose understanding and sacrifice have been exemplary. The tolerance and patience of my children, I value and appreciate greatly.

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ABSTRACT

The thesis entitled "Some Aspects of Shear and Natural Convective Flows Obeying Arrhenius Law of Viscosity" deals basically with shear flows in channels of different geometries, whose boundaries are maintained at a constant temperature and when heat generation due to viscous dissipation is taken into account. The relation between the viscosity μ and temperature T is taken as

$$\mu = \mu_0 \exp\left(\beta \left(\frac{T_0}{T} - 1\right)\right)$$

where μ_0 is the viscosity at reference temperature T_0 and β is a nondimensional activation energy parameter which measures the sensitivity of viscosity to variations in temperature.

In all the shear flow problems considered in this thesis, it is found that the shear rate-shear stress diagram is S-shaped or monotonic depending on whether the activation energy parameter is large or small. The parameter of each problem is identified and its effect on shear rate-stress diagram and heat transfer is considered.

In Chapter II the effect of radius ratio, angular velocity ratio on flow in an annulus due to the axial, circular motion of the boundaries and the effect of aspect

ratio on flow in a rectangular duct due to the motion of a moving wall is considered.

In Chapter III, the effect of permeability on flow in a porous medium between plane and cylindrical surfaces in relative motion is studied.

Chapter IV is concerned with the effect of velocity slip at the boundaries on plane and annular Couette flows. Further, the effect of the thickness of the porous lining on flow of constant viscosity fluid through a pipe and a rectangular duct whose boundaries are provided with a porous lining is considered. It is shown that for small values of the thickness of the porous lining, the effect is to increase the mass flow rate and to decrease the friction factor.

Chapter V deals with plane, annular and circular Couette flows with surface mass transfer. The effect of Cross flow Reynolds number is studied.

In Chapter VI, we have studied the effect of activation energy on buoyancy induced convection due to differential heating in fluid saturated porous enclosures of rectangular and annular geometries. It is shown that an increase in the activation energy parameter causes an increase in circulation and heat transfer rate.

CONTENTS

		Page
	NOMENCLATURE	i
CHAPTER I	INTRODUCTION	1
1.0	General Introduction	1
1.1	Heat Transfer by Convection	4
1.2	Governing Equations	10
1.2.1	Flows Through Porous Media	13
1.2.2	Boussinesq Approximation	14
1.3	Historical Account of Previous Work	15
1.3.1	Laminar Flows with Temperature Dependent Viscosity	15
1.3.2	Natural Convection in Enclosures Containing Porous Medium	23
1.4	Present Study	26
CHAPTER II	MULTIPLE SOLUTIONS AND HYSTERESIS IN SHEAR FLOWS IN ANNULAR AND RECTANGULAR CHANNELS	33
2.0	Introduction	33
2.1	Annular Channel: Effect of Radius Ratio	35
2.1.0	Introduction	35
2.1.1	Formulation of the Problem	36
2.1.2	Particular Case	39
2.1.3	Numerical Method	40
2.1.4	Discussion of Results	41
2.1.5	Conclusions	46

	Page	
2.2	Circular Couette Flow Effect of Angular Velocity Ratio	48
2.2.0	Introduction	48
2.2.1	Formulation of the Problem	49
2.2.2	Solution	51
2.2.3	Discussion of Results	52
2.2.4	Conclusions	57
2.3	Rectangular Duct:Effect of Aspect Ratio	58
2.3.0	Introduction	58
2.3.1	Formulation of the Problem	59
2.3.2	Solution	61
2.3.3	Numerical Method	63
2.3.4	Discussion of Results	64
2.3.5	Conclusions	67
CHAPTER III	MULTIPLE SOLUTIONS AND HYSTERESIS IN STEADY LAMINAR FLOWS IN A POROUS MEDIUM BETWEEN SURFACES IN RELATIVE MOTION	68
3.0	Introduction	68
3.1	Parallel Plate Channel	72
3.1.0	Introduction	72
3.1.1	Formulation of the Problem	72
3.1.2	Solution	75
3.1.3	Discussion of Results	76
3.1.4	Conclusions	80

		Page
3.2	Annular Channel	82
3.2.0	Introduction	82
3.2.1	Formulation of the Problem	82
3.2.2	Shear Stress	84
3.2.3	Numerical Solution and Results	85
3.2.4	Conclusions	89
CHAPTER IV	FLOW PAST POROUS MEDIA	90
4.0	Introduction	90
4.1	Effect of Velocity Slip on Multiple Solutions and Hysteresis in Couette Flow	93
4.1.0	Introduction	93
4.1.1	Formulation of the Problem	93
4.1.2	Solution	97
4.1.3	Results and Discussion	97
4.1.4	Conclusions	100
4.2	Effect of Velocity Slip on Multiple Solutions and Hysteresis in Shear Flow Through an Annulus	101
4.2.0	Introduction	101
4.2.1	Formulation of the Problem	101
4.2.2	Particular Case	104
4.2.3	Results and Discussion	105
4.2.4	Conclusions	107

	Page	
4.3	Effect of Slip Velocity on the Pipe Flow and Heat Transfer When the Inner Surface of the Pipe is Provided with Non-Erodible Porous Lining	109
4.3.0	Introduction	109
4.3.1	Formulation of the Problem	110
4.3.2	Solution	111
4.3.3	Mass Flux	112
4.3.4	Friction Factor	117
4.3.5	Heat Transfer by Viscous Dissipation	118
4.3.6	Conclusions	119
4.4	Flow in a Rectangular Channel when its Boundaries are Provided with a Lining of Non-Erodible Porous Material	121
4.4.0	Introduction	121
4.4.1	Formulation of the Problem	121
4.4.2	Numerical Method	124
4.4.3	Results and Discussion	125
4.4.4	Conclusions	128
CHAPTER V	EFFECT OF SURFACE MASS TRANSFER ON MULTIPLE SOLUTIONS AND HYSTERESIS IN STEADY LAMINAR FLOWS IN CHANNELS WITH POROUS WALLS	130
5.0	Introduction	130
5.1	Flow Between Parallel Porous Plates	132
5.1.0	Introduction	132
5.1.1	Formulation of the Problem	133

	Page	
5.1.2	Solution	136
5.1.3	Discussion of Results	137
5.1.4	Conclusions	141
5.2	Flow in Annulus with Porous Walls	142
5.2.0	Introduction	142
5.2.1	Formulation of the Problem	143
5.2.2	Solution	146
5.2.3	Discussion of Results	147
5.2.4	Conclusions	150
5.3	Flow Between Rotating Cylinders	151
5.3.0	Introduction	151
5.3.1	Formulation of the Problem	151
5.3.2	Particular Case	155
5.3.3	Discussion of Results	157
5.3.4	Conclusions	159
CHAPTER VI	EFFECT OF ACTIVATION ENERGY ON NATURAL CONVECTION IN POROUS CAVITIES	161
6.0	Introduction	161
6.1	Square Cavity with Isothermal Vertical Walls	163
6.1.0	Introduction	163
6.1.1	Formulation of the Problem	164
6.1.2	Numerical Method	167
6.1.3	Discussion of Results	170
6.1.4	Conclusions	175

	Page	
6.2	Square Cavity with Constant Heat Flux on One Vertical Wall	176
6.2.0	Introduction	176
6.2.1	Formulation of the Problem	176
6.2.2	Computational Details	177
6.2.3	Discussion of Results	178
6.2.4	Conclusions	183
6.3	Annular Cavity with Isothermal Vertical Walls	184
6.3.0	Introduction	184
6.3.1	Formulation of the Problem	187
6.3.2	Computational Details	189
6.3.3	Results and Discussion	190
6.3.4	Conclusions	193
	REFERENCES	195