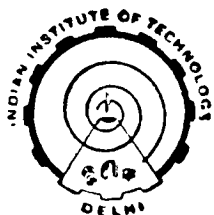


**RESONANT RESPONSE OF BEAMS AND RECTANGULAR
PLATES WITH DISCRETE VISCOELASTIC
DAMPERS AND ABSORBERS**

Thesis submitted to the
Indian Institute of Technology, Delhi
for the award of the degree of
DOCTOR OF PHILOSOPHY

by

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JULY, 1987

A B S T R A C T

The present work deals with the analysis of forced vibration of beams and rectangular plates having concentrated viscoelastic dampers and vibration absorbers.

The dynamic behaviour of various configurations under sinusoidally varying concentrated force excitation is analysed and results for peak values of displacement response and force transmissibility are obtained.

The cases of beam analysed are those of a beam carrying a concentrated mass with viscoelastic and supports to act as vibration dampers; a simply supported beam with an arbitrarily positioned intermediate viscoelastic damper and also a case with a mass supported on viscoelastic spring to act as vibration absorber, a simply supported beam with multiple viscoelastic dampers and a case of multiple vibration absorbers to control various mode resonances of the beam vibration.

The cases of plates analysed are those of a rectangular plate with a concentrated mass and supported on viscoelastic springs along its four edges to restrain its translational and rotational motion; a simply supported rectangular plate having an arbitrarily positioned concentrated viscoelastic damper and also a case with mass supported on viscoelastic spring on the plate to act as vibration absorber.

In the case of beams, the solution for transverse displacement response and force transmissibility are sought by use of transfer matrix approach. As a measure of check, two representative examples of simply supported beam with an

intermediate viscoelastic spring (damper) and beam with an absorber are also solved by the use of classical method of series summation of beam functions.

The solution in the case of rectangular plate with a concentrated mass and viscoelastic support along its four edges is sought by the series summation of polynomial coordinate functions combined with Galerkin's method.

Solutions for simply supported rectangular plate with an intermediate viscoelastic damper and for that of a plate with viscoelastic vibration absorber are obtained by the use of series summation method which is an extension of the method employed in the case of simply supported beam.

The material properties of linear viscoelastic spring are represented by complex shear modulus.

The numerical results of peak values of displacement response and force transmissibility due to sinusoidally varying concentrated force are obtained by using a digital computing system.

Peak values of transverse displacement response and force transmissibility versus stiffness ratio and material loss factor for various resonant conditions are presented. Resonant frequency ratio variations with respect to stiffness ratios and material loss factors are also presented.

It has been observed that the peak values of displacement response and force transmissibility are high at lower values of stiffness ratios and decrease with the increase of stiffness ratio and after having reached their minimum values, start increasing with further increase of the stiffness ratio

for a constant values of the material loss factor of the viscoelastic material employed for the spring. It is also seen that the peak values of displacement and force transmissibility decrease with the increase of material loss factors when the stiffness ratio is kept constant at 1.0. The resonant frequency ratios increase at a very slow rate with the increase of stiffness ratios. It is only at higher values of stiffness ratios they become perceptible (with $\eta = 0.5$ constant). Variation of resonant frequency ratio is insignificant with the variation of the material loss factor of the viscoelastic spring.

It is observed that in reducing the peak values of displacement response and force transmissibility the spring mass system(vibration absorber) is more effective as compared with those obtained by the use of viscoelastic dampers.

The multiple viscoelastic dampers and vibration absorbers employed for reducing the peak values of displacement response and force transmissibility are found to be quite effective.


Discussion and general conclusion of the present work and suggestions for further work are given at the end.

C E R T I F I C A T E

I am satisfied that the thesis entitled "Resonant Response of Beams and Rectangular Plates with Discrete Viscoelastic Dampers and Absorbers" presented by Raj Kumar Nanda is worthy of consideration for the award of the degree of 'Doctor of Philosophy' and is a record of the original bonafide research work carried out by him under my guidance and supervision and that the results contained in it have not been submitted in part or full to any other University of Institute for the award of any degree/diploma.

I certify that he has pursued the prescribed course of research.

July, 1987


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A C K N O W L E D G E M E N T S

I wish to express my profound gratitude to Prof. B.C. Nakra, of the Department of Mechanical Engineering, Indian Institute of Technology, New Delhi, for suggesting the problem and supervising the work on this thesis. His inspiring guidance, valuable discussions and constant encouragement helped me to accomplish this work.

I wish to express my indebtedness to the authorities of the Department of Mechanical Engineering, I.I.T. Delhi, for extending various facilities to complete this work. Thanks are also due to the staff of the Computer Centre for their help and cooperation.

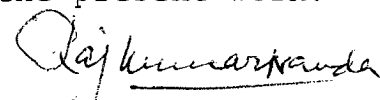
Appreciation is also extended to the authorities of Madhav Institute of Technology and Science, Gwalior-474 005 and to the Quality Improvement Programme for giving the leave and scholarship to carry-out the research work at I.I.T., Delhi.

Special words of thanks are due to Prof. H.B. Mathur and Prof. K.N. Gupta, of the Department of Mechanical Engg., I.I.T., Delhi, for their moral support.

I extend my words of appreciation to my colleagues of MITS, Gwalior, for bearing the extra teaching load during my absence from the Institute for this work.

Finally, I offer my heartfelt gratitude to the members of my family for patiently enduring the pains and difficulties which resulted from my absence for the present work.

July, 1987


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