GENERALIZED MODEL BASED FRAMEWORK AND APPLICATIONS

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INDIAN INSTITUTE OF TECHNOLOGY DELHI

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by

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I want to dedicate this endeavour to my parents, my grandparents and my teachers—my mother for teaching us how to work hard, my father for teaching us how to fly, my grandparents for giving us the roots, and my teachers for giving us the direction in life. Our education has been their only worldly wealth.

THESIS CERTIFICATE

This is to certify that the thesis titled **Generalized Model Based Framework and Applications**, submitted by **Shruti Sharma**, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by her under our supervision. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

Real-world data involving multivariate observations exhibit skewness, correlated entries across various dimensions and non-Gaussian characteristics such as low-dimensional structures with densities that are sharply peaked and possess heavy tails. As a result, the normality assumption is violated and it is desirable to have methods to model a broader class of signals exploiting domain specific knowledge, while offering simple solutions such as available with Gaussian models.

This thesis, *Generalized Model-Based Frameworks and Applications*, attempts to address the problem of data modeling under model-based framework, when data exhibit correlation structure with heavy tails and skeweness. The basic idea is to assume specific models for data generation, so that we can have specialized algorithms, that require less data for inference. In this work, we rely on the generalized scale mixture model, which comprises of various random and deterministic parameters. By varying distribution over random parameter, we derive a family of generalized multivariate heavy tailed distributions, which encompasses several choice of classical distributions such as Laplace, Student's t and Jeferrys prior. For inference, we develop a hybrid framework, where random parameters are estimated using Variational Bayes (VB) and deterministic parameters using Expectation Maximization (EM). To show the applicability of the proposed model, we present its utility in two different application areas.

Firstly, we consider the model-based compressed sensing framework associated with Bayesian linear model. Specifically, we address the Bayesian group sparse modeling with correlated entries and derive cSVB (correlated Sparse Variational Bayes) framework. We also propose two frameworks, *viz.* TSVB (Temporal Sparse Variational Bayes) and TMSVB (low complexity version of TSVB), to solve related problems on sparse signal recovery in the multiple measurement vector (MMV) context. Next, we consider its extension to spatio-temporal data and present the Spatio-Temporal Sparse Variaitonal Bayes (STSVB) framework. The proposed algorithm processes multi-dimensional data, and is particularly helpful where we have a limited access to compuational resources. We briefly discuss applications in the domains of fetal electrocardiogram (FECGs), electroencephalogram (EEG) data processing, and in the modeling of a dynamic hand dataset. We also present the utility of cSVB and STSVB frameworks for Steady-State Visual Evoked Potential (SSVEP) based EEG signals, where the task is to detect frequency components present in the data.

Next, we discuss a novel model-based clustering/classification framework using a gener-

alized class of skewed distribution. Specifically, we present an alternative to the Gaussian Mixture Model (GMM), based on the EM framework, that is capable of handling more complex structures present in data, while offering advantages of almost similar computational complexity as that of GMM. Experimental results on five real-world datasets demonstrate the effectiveness of the model.

Finally, we explore the possible future directions in the context of model-based methods.

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सार

बहुभिन्नरूपी प्रेक्षणों से जुड़े वास्तविक-विश्व डेटा में विषमता, विभिन्न आयामों में सहसंबद्ध प्रविष्टियाँ और गैर-गॉसियन विशेषताओं जैसे निम्न-आयामी संरचनाएँ घनत्व के साथ प्रदर्शित होती हैं जो तेजी से चरम पर होती हैं और जिनमें भारी पूंछ होती है। नतीजतन, सामान्यता धारणा का उल्लंघन किया जाता है और यह वांछनीय है कि गाऊसी मॉडल के साथ उपलब्ध सरल समाधानों की पेशकश करते हुए डोमेन विशिष्ट ज्ञान का शोषण करने वाले संकेतों के व्यापक वर्ग को मॉडल करने के तरीके हैं।

यह थीसिस, सामान्यीकृत मॉडल-आधारित ढांचे और अनुप्रयोग, मॉडल-आधारित ढांचे के तहत डेटा मॉडलिंग की समस्या को हल करने का प्रयास करते हैं, जब डेटा भारी पूंछ और तिरछीपन के साथ सहसंबंध संरचना प्रदर्शित करता है। मूल विचार डेटा पीढ़ी के लिए विशिष्ट मॉडल ग्रहण करना है, ताकि हमारे पास विशेष एल्गोरिदम हो सकें, जिसके लिए अनुमान के लिए कम डेटा की आवश्यकता होती है। इस काम में, हम सामान्यीकृत पैमाने के मिश्रण मॉडल पर भरोसा करते हैं, जिसमें विभिन्न यादृच्छिक और नियतात्मक पैरामीटर शामिल हैं। यादृच्छिक पैरामीटर पर वितरण अलग-अलग करके, हम सामान्यीकृत बहुभिन्नरूपी भारी पूंछ वाले वितरण का एक परिवार प्राप्त करते हैं, जिसमें शास्त्रीय वितरण के कई विकल्प शामिल हैं जैसे कि लैपलेस, स्टूडेंट का टी और जेफरी का पूर्व। अनुमान के लिए, हम एक हाइब्रिड फ्रेमवर्क विकसित करते हैं, जहां वैरिएशनल बेयस (VB) और नियतात्मक मापदंडों का उपयोग करके एक्सपेक्टेशन मैक्सिमाइजेशन (EM) का उपयोग करके यादृच्छिक मापदंडों का अनुमान लगाया जाता है। प्रस्तावित मॉडल की प्रयोज्यता दिखाने के लिए, हम दो अलग-अलग अनुप्रयोग क्षेत्रों में इसकी उपयोगिता प्रस्तुत करते हैं।

सबसे पहले, हम बायेसियन रैखिक मॉडल से जुड़े मॉडल-आधारित संपीड़ित संवेदन ढांचे पर विचार करते हैं। विशेष रूप से, हम सहसंबद्ध प्रविष्टियों के साथ बायेसियन समूह विरल मॉडलिंग को संबोधित करते हैं और cSVB (correlated Sparse Variational Bayes) ढांचे को प्राप्त करते हैं। हम दो रूपरेखाओं का भी प्रस्ताव करते हैं, अर्थात। TSVB (Temporal Sparse Variational Bayes) और TMSVB (TSVB का एक कम जटिलता संस्करण), मल्टीपल मेजरमेंट वेक्टर (MMV) संदर्भ में विरल सिग्नल रिकवरी पर संबंधित समस्याओं को हल करने के लिए। इसके बाद, हम spatiotemporal डेटा के लिए इसके विस्तार पर विचार करते हैं और Spatio-temporal Sparse Variational Bayes (STSVB) फ्रेमवर्क प्रस्तुत करते हैं। प्रस्तावित एल्गोरिदम बहु-आयामी डेटा को संसाधित करता है और विशेष रूप से सहायक होता है जहां हमारे पास कम्प्यूटेशनल संसाधनों तक सीमित पहुंच होती है। हम संक्षेप में भ्रूण इलेक्ट्रोकार्डियोग्राम (FECGs), इलेक्ट्रोएन्सेफलोग्राम (EEG) डेटा प्रोसेसिंग के डोमेन में और एक गतिशील हाथ डेटासेट के मॉडलिंग में अनुप्रयोगों पर चर्चा करते हैं। हम की उपयोगिता भी प्रस्तुत करते है| स्थिर-राज्य दृश्य विकसित संभावित (SSVEP) आधारित EEG संकेतों के लिए cSVB और STSVB ढांचे, जहां कार्य डेटा में मौजूद आवृत्ति घटकों का पता लगाना है।

इसके बाद, हम विषम वितरण के एक सामान्यीकृत वर्ग का उपयोग करते हुए एक उपन्यास मॉडल-आधारित क्लस्टरिंग/वर्गीकरण ढांचे पर चर्चा करते हैं। विशेष रूप से, हम ईएम ढांचे के आधार पर गाऊसी मिश्रण मॉडल (GMM) का एक विकल्प प्रस्तुत करते हैं, जो डेटा में मौजूद अधिक जटिल संरचनाओं को संभालने में सक्षम है, जबकि जीएमएम के लगभग समान कम्प्यूटेशनल जटिलता के फायदे पेश करते हैं। पांच वास्तविक दुनिया के डेटासेट पर प्रायोगिक परिणाम मॉडल की प्रभावशीलता को प्रदर्शित करते हैं।

अंत में, हम मॉडल-आधारित विधियों के संदर्भ में भविष्य की संभावित दिशाओं का पता लगाते हैं|

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ABBREVIATIONS

AIS	Australian Institute of Sports
AR	Autoregressive
ARD	Automatic Relevance Determination
ARI	Adjusted Random Index
BCI	Brain Computer Interface
BIC	Bayesian Information Criterion
BSBL	Block Sparse Bayesian Learning (Block SBL)
CAR	Conditional Autoregressive
CCA	Canonical Correlation Analysis
CLT	Central Limit Theorem
CR	Compression Ratio
CS	Compressed Sensing
cSVB	correlated Sparse Variational Bayes
cJSVB	cSVB based on Jeffreys prior
cLSVB	cSVB based on Laplace distribution
cStSVB	cSVB based on Student distribution
DCT	Discrete Cosine Transform
DDA	Descriptive Discriminant Analysis
DOA	Direction of Arrival
EEG	Electroencephalogram
EPGMM	Expanded Parsimonious Gaussian mixture model
EM	Expectation Maximization
FBCCA	Filter Bank Canonical Correlation Analysis
FECG	Fetal electrocardiogram
FM	Finite Mixture Model
FM-HMT	FM with Hierarchical Multivariate Student
FM-MGH	FM with Multivariate Generalized Hyperbolic
FM-MN	FM with Multivariate Normal
FM-MT	FM with Multivarite Student
FM-MNIG	FM with Multivariate Normal Inverse Gaussian
FM-rMSN	FM with restricted Multivariate Skewed Normal
FM-rMST	FM with restricted Multivariate Skewed Student
FM-rMSTN	FM with restricted Multivariate Skewed Student Normal
FM-uMST	FM with un-restricted Multivariate Skewed Student

FOCUSS	FOCal Underdetermined System Solution
GH	Generalized Hyperbolic
GIG	Generalized Inverse Gaussian
GMM	Gaussian Mixture Model
GSM	Gaussian Scale Mixture
ICA	Indpendent Component Analysis
L1MCCA	L1-regularized Multiway CCA
MANOVA	Multivariate Analysis of Variance
MCGFA	Mixture of Contaminated Gaussian Factor Analyzers
MCMC	Markov Chain Monte Carlo
MCR	Misclassification Rate
MEG	Magnetoencephalography
ML	Machine Learning
MMtFA	Mixtures of Modified t-factor Analyzers
MMV	Multiple Measurement Vector
MSE	Mean Square Error
MSNIG	Multivariate Skewed Normal Inverse Gaussian
MSSt	Multivariate Skewed Student
NIG	Normal Inverse Gaussian
NVMM	Normal Variance Mean Mixture
PCA	Principal Component Analysis
PSNR	Peak Signal to Noise Ratio
OMP	Orthogonal Matching Pursuit
RIP	Restricted Isometry Property
SAL	Shifted Asymmetric Laplace
SBL	Sparse Bayesian Learning
SMV	Single Measurement Vector
SSR	Sparse Signal Recovery
SSVEP	Steady State Visual Evoked Potential
STSBL	Spatio-Temporal SBL
STSVB	Spatio-Temporal Sparse Variational Bayes
STSVB-J	STSVB based on Jeffreys as marginal
STSVB-L	STSVB based on Laplace as marginal
STSVB-St	STSVB based on Student's t as marginal
SVB	Sparse Variational Bayes
TRCA	Task Related Component analysis
TMSVB	Low Complexity version of Temporal (Multiple) Sparse Variational Bayes
TSVB	Temporal Sparse Variational Bayes
JTMSVB	TMSVB based on Jeffreys as Marginal
JTSVB	TSVB based on Jeffreys as Marginal

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LTMSVB	TMSVB based on Laplace as Marginal
LTSVB	TSVB based on Laplace as Marginal
StTSVB	TMSVB based on Student as Marginal
StTSVB	TSVB based on Student as Marginal
TSBL	Temporal SBL
VAE	Variational Autoencoders
VB	Variational Bayes
VG	Variance Gamma

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NOTATION

Small letters	Scalars
Small bold letters	Vectors
Capital bold letters	Matrices
$\text{diag}(a_1, a_2, \ldots, a_n)$	Diagonal matrix with principal diagonal $a_1, a_2, \ldots, a_n$
$\text{diag}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n)$	Block diagonal matrix with principal diagonal $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n$
$\mathbf{A}_{.j}$	j^{th} column of matrix $\mathbf{A}$
$\mathbf{A}_{i.}$	i^{th} row of matrix $\mathbf{A}$
$\mathbf{A}_{[i].}$	ith block of all columns
$\mathbf{A}_{[i]}$	ith diagonal block
$\mathbf{A}_{[i]\ell}$	ℓth column of $\mathbf{A}$ in ith block
a_{ij}	ij^{th} element of matrix $\mathbf{A}$
$\det(\mathbf{A})$ or $\lvert\mathbf{A}\rvert$	Determinant of $\mathbf{A}$
$\mathbf{A}^t$	Transpose of $\mathbf{A}$
$\lVert\mathbf{x}\rVert_2$	l_2 norm of $\mathbf{x}$ $\left(= \sqrt{\sum_i x_i^2}\right)$
$\lVert\mathbf{x}\rVert_\infty$	l_∞ norm of $\mathbf{x}$ $\left(= \max_i x_i\right)$
$\lVert\mathbf{A}\rVert_{\mathcal{F}}$	Frobenius norm of $\mathbf{A}$ $\left(= \sqrt{tr(\mathbf{A}\mathbf{A}^t)} = \sqrt{tr(\mathbf{A}^t\mathbf{A})}\right)$
$\text{tr}(\mathbf{A})$	Trace of matrix $\mathbf{A}$
$\text{vec}(\mathbf{A})$	Vectorization of $\mathbf{A}$
$\mathbf{A} \otimes \mathbf{B}$	Kronecker Product of matrices $\mathbf{A}$ and $\mathbf{B}$
$\mathbb{E}[.]$	Expected value of random parameter
$\mathcal{N}(.)$	Gaussian distribution
$\mathcal{G}(.)$	Gamma distribution
$\mathcal{IG}(.)$	Inverse Gamma Distribution
$\mathcal{GIG}(.)$	Generalized Inverse Gaussian distribution
$\Gamma(.)$	Gamma function