APPLICATION OF INTEGRAL METHOD TO DARCIAN AND NON-DARCIAN NATURAL CONVECTION ACROSS NON-ISOTHERMAL SURFACES IMMERSED IN INHOMOGENEOUS POROUS MEDIA

by

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THESIS SUBMITTED
IN FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI
INDIA

JANUARY, 1993

CERTIFICATE

This is to certify that the thesis entitled "**APPLICATION OF INTEGRAL METHOD TO DARCIAN AND NON-DARCIAN NATURAL CONVECTION ACROSS NON-ISOTHERMAL SURFACES IMMERSED IN INHOMOGENEOUS POROUS MEDIA**" which is being submitted by Miss Shobha Sood to the Indian Institute of Technology, Delhi, for the award of the degree of DOCTOR OF PHILOSOPHY in Mathematics, is a record of bonafide research work carried out by her under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENTS

I take this opportunity to express my sincere gratitude to Prof. K.N. Mehta, Head, Department of Mathematics, I.I.T. Delhi, for his consistent guidance, support and supervision throughout the preparation of this thesis.

I owe my sincere thanks to the authorities of I.I.T. Delhi and Head of the Computer Service Centre for the research facilities provided to me.

My profound thanks are also due to the faculty members of the Department of Mathematics, I.I.T. Delhi for the courses taught to me at the M.Sc. and Pre-Ph.D. levels which have proved to be helpful in my research work. I am also indebted to all my friends at I.I.T. Delhi for their constant help throughout the tenure of my research work.

Lastly to my parents and brother Deepak I owe more than I can possibly express for their love, encouragement and sacrifices which have been instrumental in shaping my career.

I.I.T. DELHI

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ABSTRACT

This thesis entitled "**APPLICATION OF INTEGRAL METHOD TO DARCIAN AND NON - DARCIAN NATURAL CONVECTION ACROSS NON - ISOTHERMAL SURFACES IMMERSED IN INHOMOGENEOUS POROUS MEDIA**", presents an evaluation of the range of application, accuracy and usefulness of integral methods in natural convection flows in porous media. The study reveals that integral method may be utilized to obtain approximate answers to free convection problems whenever exact analytical solutions become too involved. Specific flow examples considered here (steady and transient natural convection across impermeable partition, vertical and horizontal plates and non-Darcy free convection) confirm its adaptability to complicated configurations. The solutions obtained suggest that the overall accuracy of the method is good. Moreover, the integral method of solution can be easily extended to incorporate variable properties, for instance permeability of the porous medium and viscosity of the fluid, variable wall temperature and discontinuities in wall temperature. In the last chapter of this dissertation an attempt is made to obtain similarity solutions for some buoyancy induced flows in porous medium. It is illustrated that similarity solutions could be obtained for only a few special cases of flows in inhomogeneous porous media.

This thesis is divided into six chapters. The first Introductory chapter gives a brief account of free convection through porous media, the basic governing equations and the

characterization of inhomogeneities and temperature dependent viscosity.

The second chapter provides an investigation of steady state Darcian flow over non-isothermal bodies. The first two sections of the chapter deal with the effect of spatial variations in permeability on the heat transfer rate while in the third section, the effect of spatial discontinuity on wall temperature of a vertical and horizontal plate on the heat transfer rate is studied.

An analysis of the transient free convection across a non-isothermal vertical and horizontal plate is made in the third chapter. The non-dimensional spatial inhomogeneity is considered to have a linear dependence on space variable where as an exponential choice is assumed for temporal inhomogeneities of the porous matrix. The viscosity is assumed to have an inverse linear dependence on temperature. The results suggest that the time required to attain steady state decreases with increase in the permeability of the porous matrix and viscosity of the fluid.

Chapter four analyses the steady state and transient development of the conjugate boundary layers due to buoyancy induced circulations and its effect on the transfer of heat across both vertical and horizontal partitions. An exponential choice for the temperature field is made. On employing Karman - Pohlhausen integral method, the governing equations for the case of a vertical partition reduce to a system of coupled non - linear

hyperbolic equations. The equation is solved analytically / numerically to yield the desired results. On the other hand, for the case of a horizontal partition, an exponential choice for the velocity field is also made along with the temperature field. The governing equations then reduce to a system of coupled integro - differential equation. The wall thermal resistance parameter ω is introduced to study the effect of varying wall thickness. It is noticed that the centrosymmetry property of the problem is destroyed by introducing spatial inhomogeneities and variable viscosity.

Non-Darcian effects on buoyancy induced flows are studied in chapter five. The flat surfaces are assumed to be permeable. To account for different mass flux across the plate, three representative values of the mass flux parameter v_0 are considered, namely, $v_0 = -1, 0, 1$. The result exhibit that the inertia of the porous matrix delays the onset of steady state.

The last chapter provides similarity solutions for buoyancy induced flows in inhomogeneous porous medium. Power law as well as linear variations in permeability are considered. The study reveals that similarity solutions for Darcy and non - Darcy flow across vertical and horizontal plates can be obtained if a power law distribution is assumed for the wall temperature. In case of linear variations in permeability similarity solutions can be obtained only for Darcy free convection flow across an isothermal vertical plate.

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