

A THESIS ON
DIRECT METHODS FOR THE NUMERICAL SOLUTION OF SECOND ORDER
INITIAL VALUE PROBLEMS AND THEIR STABILITY ANALYSIS

By
SARJIT RAI SHARMA
Department of Mathematics
Indian Institute of Technology
New Delhi

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CERTIFICATE

This is to certify that the thesis entitled "Direct Methods for the Numerical Solution of Second Order Initial Value Problems and Their Stability Analysis" which is being submitted by Mr. Sarjit Rai Sharm for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Hauz Khas, New Delhi, is a record of bonafide research work. He has worked for the last three and a half years under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

**(M.M. Chowla)
Professor
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-29
India**

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