

SYMMETRY AWARE INFERENCE AND DECISION MAKING IN PROBABILISTIC MODELS

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SYMMETRY AWARE INFERENCE AND DECISION MAKING IN PROBABILISTIC MODELS

by

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Certificate

This is to certify that the thesis titled **Symmetry Aware Inference and Decision Making in Probabilistic Models** being submitted by **Mr. Ankit Anand** for the award of **Doctor of Philosophy** in Department of Computer Science and Engineering is a record of bona fide work carried out by him under my guidance and supervision at the **Department of Computer Science and Engineering, Indian Institute of Technology Delhi**. The work presented in this thesis has not been submitted elsewhere, either in part or full, for the award of any other degree or diploma unless otherwise stated explicitly. In particular, work done in Chapters 3, 4, 7, 8 and 9 were done jointly with undergraduate students. Work done in Chapter 5 was done jointly with a master's student. In each case, the part done by the collaborators appeared in their respective bachelor's or master's theses.

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Acknowledgements

“No dream is ever chased alone.”

Rahul S. Dravid, Indian Cricketer

As I stand near the end of my journey of formal education, I would like to look back and express thanks to some really great people whom I met on the way and without whom, this long path would have looked really dull.

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Abstract

Many contemporary artificial intelligence algorithms fail to scale to large problems, because the problem sizes typically increase exponentially with the number of features. Even though a problems size may be too large, there often exist repeated sub-structures, resulting in symmetries or other kinds of invariances within the problem. These symmetries and invariances, when identified accurately, can substantially save on downstream computation time. This thesis proposes the idea of symmetry aware AI, in which AI and ML algorithms compute these symmetries, and then use them for improving their efficiency and performance. Specifically, this thesis studies symmetry aware AI in the context of three different AI problems: (i) probabilistic inference in probabilistic graphical models, (ii) sequential decision making under uncertainty and (iii) structured output prediction in computer vision.

First, in probabilistic inference, we study different types of state symmetries to help speedup marginal inference. Specifically, we define novel notions of contextual symmetries [Anand et. al., IJCAI 16], variable-value and non-equicardinal symmetries [Anand et. al., AISTATS 18] and block-value symmetries [Madan et. al. UAI 2018], and propose algorithms to compute these. Further, we incorporate these symmetries for improving the mixing time of Markov Chain Monte Carlo (MCMC) methods.

Second, we define the ASAP (Abstraction of State-Action Pairs) framework, which extends and unifies past work on domain abstractions in sequential decision making under uncertainty. It holistically aggregates both states and state-action pairs, thereby, identifying significantly more symmetries than previous work. We also propose two novel algorithms: ASAP-UCT [Anand et. al., IJCAI 15] and OGA-UCT [Anand et. al., ICAPS 16], which use ASAP symmetries within a UCT framework, thus, combining strengths of online planning with domain abstractions.

Last but not the least, we explore the use of symmetries in state-of-the-art algorithms for real-world computer vision problems. We propose a novel coarse-to-fine symmetry aware template [Habeeb et. al., IJCAI 17] to exploit symmetries in structured output prediction tasks of stereo-vision and image segmentation. Our approximate notion of top-k label heuristic is robust across problems and provide significant time gains in near state-of-the-art algorithms for these tasks.

सार

प्रोबेबिलिस्टिक मॉडल्स में सिमिट्री अवेयरनेस अनुमान एंड डिसीजन मेकिंग

कई समकालीन आर्टिफिशियल इंटेलिजेंस (ए.आई.) (अर्थात कृत्रिम बुद्धि) एल्गोरिदम बड़ी समस्याओं के पैमाने पर विफल होते हैं, क्योंकि समस्या के आकार आमतौर पर विशेषताओं की संख्या के साथ तेजी से बढ़ते हैं। भले ही किसी समस्या का आकार बहुत बड़ा हो, लेकिन अक्सर उप-संरचनाएँ मौजूद रहती हैं, जिसके परिणामस्वरूप समस्या के भीतर समरूपता होती है। इन समरूपताओं की जब सही पहचान की जाती है, तो डाउनस्ट्रीम गणना समय पर काफी बचत हो सकती है। यह थीसिस समरूपता के बारे में जागरूक ए.आई. के विचार का प्रस्ताव करती है, जिसमें ए.आई. और एम.एल. एल्गोरिदम इन समरूपताओं की गणना करते हैं, और फिर उनकी दक्षता और प्रदर्शन में सुधार के लिए उनका उपयोग करते हैं। विशेष रूप से, यह थीसिस तीन अलग-अलग एआई समस्याओं के संदर्भ में एआई को समरूपता से अवगत कराती है: (i) संभाव्य ग्राफिकल मॉडल में संभाव्यता का अनुमान, (ii) अनिश्चितता के तहत क्रमिक निर्णय और (iii) कंप्यूटर विज्ञान में संरचित आउटपुट भविष्यवाणी।

सबसे पहले, संभाव्य निष्कर्ष में, हम मार्जिनल अनुमान में मदद करने के लिए विभिन्न प्रकार के अवस्था समरूपता का अध्ययन करते हैं। विशेष रूप से, हम संदर्भ-गत सिमिट्रीज़ [आनंद एट अल., IJCAI '16], की नई धारणाओं को परिभाषित करते हैं। वैरिएबल-वैल्यू और नॉन-इक्विवार्डिनल सिमिट्रीज़ [आनंद एट अल., AISTATS '18] और ब्लॉक-वैल्यू समरूपता [मदान एट अल. UAI 2018], और इनकी गणना के लिए एल्गोरिदम का प्रस्ताव करते हैं। इसके अलावा, हम मार्कोव चेन मॉंटे कार्लो (MCMC) विधियों के मिश्रण समय में सुधार के लिए इन समरूपताओं को शामिल करते हैं।

दूसरा, हम ASAP (स्टेट-एक्शन पेयर की एक्सट्रैक्ट) रूपरेखा को परिभाषित करते हैं, जो अनिश्चितता के तहत क्रमिक निर्णय लेने में डोमेन अमूर्त पर पिछले काम का विस्तार और एकीकरण करता है। यह दोनों स्टेट (अवस्था) और स्टेट-एक्शन जोड़े को समग्र रूप से एकत्रित करता है, जिससे पिछले कार्य की तुलना में काफी अधिक समरूपता की पहचान होती है। हम दो नए एल्गोरिदम का प्रस्ताव भी देते हैं: एसएपी-यूसीटी [आनंद एट अल., IJCAI '15] और OGA-UCT [आनंद एट अल., ICAPS '16], जो एक UCT ढांचे के भीतर ASAP समरूपता का उपयोग करते हैं, इस प्रकार, डोमेन सार के साथ ऑनलाइन योजना की ताकत का संयोजन करते हैं।

अंतिम लेकिन कम से कम नहीं, हम वास्तविक दुनिया की कंप्यूटर दृष्टि समस्याओं के लिए अत्याधुनिक एल्गोरिदम में समरूपता के उपयोग का पता लगाते हैं। हम एक मोटे-से-ठीक समरूपता वाले जागरूक टेम्पलेट [हबीब एट अल., IJCAI '17] का प्रस्ताव करते हैं। स्टीरियो-दृष्टि और छवि विभाजन के संरचित आउटपुट भविष्यवाणी कार्यों में समरूपता का फायदा उठाने के लिए टॉप-के लेबल हेयुरिस्टिक की हमारी अनुमानित धारणा समस्याओं के प्रति मजबूत है और इन कार्यों के लिए अत्याधुनिक एल्गोरिदम के पास महत्वपूर्ण समय लाभ प्रदान करती है।

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