CRACK GROWTH INITIATION AND PROPAGATION DUE TO A CLOSING PORE IN A NATURAL QUASI-BRITTLE ORTHOTROPIC SOLID

SAILENDU BISWAL

DEPARTMENT OF APPLIED MECHANICS
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by

Sailendu Biswal

Department of Applied Mechanics

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CERTIFICATE

This is to certify that the thesis entitled **"Crack growth initiation and propagation due to a closing pore in a natural quasi-brittle orthotropic solid"**, submitted by **Sailendu Biswal** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Applied Mechanics, is a record of the original, bonafide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

Prof. Gaurav Singh
Associate Professor
Department of Applied Mechanics
Indian Institute of Technology Delhi
New Delhi - 110016
India

Date: 09/05/2024

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ABSTRACT

This thesis discusses the mechanics of crack formation, the initiation of its growth and eventual propagation leading to fracture due to a closing pore in pine wood (*Pinus radiata*). A closing pore situation begins with the compression of a specimen with an open pore which gradually closes by pore surfaces coming into contact. Thereafter, cracks form near the pore which grows and eventually fracture the specimen. The combined action of contact and fracture mechanics due to a closing pore is the first novelty addressed in this thesis using experiments and FEM simulations.

The simulations of closing pore situation in wood requires the determination of elastic and fracture properties of wood. This is extremely more challenging compared to synthetic materials due to natural occurrence, quasi-brittle nature and macroscopic orthotropy of wood. Experiments have been conducted on wood samples to determine it's elastic properties in tension, shear and compression. Thereafter, fracture properties like critical energy release rate and traction-separation relations have been experimentally determined for mode I and II using multi-specimen approach. The use of multi-specimen approach for calculating the fracture toughness of wood is the second novelty of this work.

Thereafter, experiments are conducted for wood samples containing central pores along the grain direction. The pore/grain orientation and surface roughness between pore surfaces are two parameters which are studied using overall mechanical response. Finally, FEM simulations are conducted for closing pore situation in pine wood by incorporating its elastic and fracture properties. Several observations from experiments are reproducible using these simulations under the constraint of linear elasticity.

The findings of this thesis can be extended to other naturally occurring quasi-brittle orthotropic solids like bone.

सार

यह शोध देवदार की लकड़ी (पाइनस रेडिएटा) में बंद होने वाले छिद्र के कारण दरार बनने की प्रक्रिया, उसके बढ़ने की प्रारंभिक प्रक्रिया, और अंततः फ्रैक्चर की वजह पर चर्चा करती है। छिद्र बंद होने की स्थिति एक खुले छिद्र वाले ठोस पदार्थ के संपीड़न से शुरू होती है, जो धीरे-धीरे संपर्क में आने वाली छिद्र सतहों से बंद हो जाती है। इसके बाद, छिद्र के पास दरारें बन जाती है जो बढ़ती हैं और अंततः नमूने को फ्रैक्चर कर देती है। बंद होते छिद्र के कारण संपर्क और फ्रैक्चर यांत्रिकी की संयुक्त क्रिया प्रयोगों और FEM सिमुलेशन का उपयोग, इस शोध में संबोधित पहली नवीनता है।

लकड़ी में छिद्रों को बंद करने की स्थिति के सिमुलेशन के लिए लकड़ी के इलास्टिक और फ्रैक्चर गुणों को निर्धारित करने की आवश्यकता होती है। प्राकृतिक, क्वासी-ब्रिटल गुण और लकड़ी की मैक्रोस्कोपिक ऑर्थोट्रॉपी के कारण कृत्रिम पदार्थ की तुलना में यह बेहद चुनौतीपूर्ण है। तनाव, कतरनी और संपीड़न में इसके इलास्टिक गुणों को निर्धारित करने के लिए लकड़ी के नमूनों की परीक्षा करी गई है। इसके बाद, बहु-नमूना पद्धति का उपयोग करके मोड *I* और *II* के लिए क्रिटिकल एनर्जी रिलीज़ रेट और ट्रैक्शन-सेपरेशन संबंधों जैसे फ्रैक्चर गुणों की प्रयोगात्मक रूप से निर्धारित किया गया है। लकड़ी की फ्रैक्चर कठोरता को निर्धारित करने के लिए बहु-नमूना पद्धति का उपयोग इस शोध की दूसरी नवीनता है।

इसके बाद, फाइबर दिशा के साथ केंद्रीय छिद्रों वाले लकड़ी के नमूनों के लिए संपीड़न प्रयोग किआ गया है। छिद्र/फाइबर की दिशा और छिद्र सतहों के बीच खुरदरापन दो मापदंड है जिनका अध्ययन संपूर्ण यांत्रिक प्रतिक्रिया का उपयोग करके किया गया है। अंततः देवदार की लकड़ी के इलास्टिक और फ्रैक्चर गुणों को उपयोग करके उसमें छिद्रों को बंद करने के लिए FEM सिमुलेशन किया गया है । लीनियर इलास्टिसिटी के आधार पर, इन सिम्युलेशनों का उपयोग करके प्रयोगों से प्राप्त नतीजों को दोहराया गया है।

इस शोध के निष्कर्षों को हड्डी जैसे अन्य प्राकृतिक रूप से पाए जाने वाले क्वासी-ब्रिटल ऑर्थोट्रोपिक ठोस पदार्थों तक विस्तारित किया जा सकता है।

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Abbreviations

ASTM	American society for testing and materials
BEM	Boundary element method
CBBM	Compliance based beam method
COD	Crack opening displacement
CTOD	Crack tip opening displacement
CZM	Cohesive zone modeling
DCB	Double cantilever beam
DDM	Displacement discontinuity method
dG	discontinuous Galerkin
DIC	Digital image correleation
ENF	End notch flexure
FEM	Finite element method
FPZ	Fracture process zone
LEFM	Linear elastic fracture mechanics
SEM	Scanning electron microscope
SENB	Single edge notch bend
SENT	Single edge notch tension
TDCB	Tapered cantilever beam
VCF	Vertebral compression fracture
WST	Wedge splitting test
XFEM	Extended finite element method

Symbols

a	Crack length
a_{eq}	Equivalent crack length
a_{FPZ}	Size of the crack inside FPZ
a_0	Initial crack length
B	Thickness of specimen
B_{kl}	Compliance matrix in plane strain
C	Compliance
C_j	Unloading compliance without crack face interference
$C_j^{'}$	Unloading compliance with crack face interference
$C(a)$	Compliance as function of crack length
E_I	Equivalent elastic modulus in Mode I
E_{II}	Equivalent elastic modulus in Mode II
E_L E_R E_T	Elastic modulus along longitudinal, radial and tangential directions respectively
f_t	Cohesive strength
G_c	Critical energy release rate or fracture toughness
G_f	Fracture energy
G_F	Total fracture energy
G_{init}	Initiation fracture energy
G_I	Energy release rate in Mode I
G_{Ic}	Critical energy release rate or fracture toughness in Mode I
G_{IF}	Total Fracture energy in Mode I
G_{II}	Energy release rate in Mode II
G_{IIc}	Critical energy release rate or fracture toughness in Mode II
G_{IIF}	Total Fracture energy in Mode II
G_{prop}	Propagation fracture energy
G_{tip}	Local crack tip energy release rate

G_T	Total energy release rate (Mode I and II)
H	Tang or supported leg of CS specimen
J	J integral
J_{Ic}	Critical value of J integral
J_{tip}	Local crack tip J integral
K_{Ic}	Critical stress intensity factor in Mode I
K_{IIc}	Critical stress intensity factor in Mode II
k_I,k_{II}	Constants of proportionality
l_{ch}	Characteristic length
l_{cz}	Cohesive zone length
m_1, m_2	Mode mix ratios
P	Load
P_c	Critical load
S_{ij}	Compliance matrix
t	Time
u	Load-line displacement
U	Airy's stress function
u_c	Load-line displacement corresponding to the critical load
u_f	Load-line displacement at complete fracture
u_{max}	Maximum value of load-line displacement
w	Crack sliding displacement
w_c	Critical crack sliding displacement
W	Width of specimen
W_F	Work of fracture
δ	Crack tip opening displacement
δ_{max}	Critical crack opening displacement
σ_n	Cohesive normal traction
σ_t	Cohesive shear traction
ν_{LR}, ν_{LT},ν_{TR}	Poisson's ratios
ϕ_1, ϕ_2	Phase angle in Mode I and II
η, κ	Dimensionless parameters in terms of material elastic constants