

ON THE DESIGN OF ANALOG & DIGITAL DIFFERENTIATORS

A thesis submitted to the
Indian Institute of Technology, Delhi
for the award of the degree of

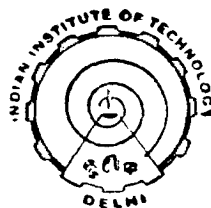
DOCTOR OF PHILOSOPHY

in

Electrical Engineering

by

BALBIR KUMAR,



DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
NEW DELHI-110016, INDIA

FEBRUARY 1988

D E D I C A T E D

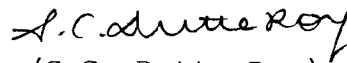
T O

M Y P A R E N T S

C E R T I F I C A T E

This is to certify that the thesis entitled, " ON THE DESIGN OF ANALOG AND DIGITAL DIFFERENTIATORS", being submitted by Balbir Kumar to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy is a record of bona fide research work carried out by him under my supervision and guidance and that in my opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.



(S.C. Dutta Roy)

Professor of Electrical Engineering,
Indian Institute of Technology, Delhi,
New Delhi-110016, INDIA

A C K N O W L E D G E M E N T

It is indeed a rare privilege for me for having won the tutelage of Professor S.C.Dutta Roy. I have been immensely benefitted by his valuable suggestions, scholarly deliberations and saintly seance. It is with great pleasure that I express my deep sense of gratitude to Professor S.C. Dutta Roy for his able guidance and constant encouragement throughout this work.

I am also grateful to the authorities of the Indian Institute of Technology, Delhi for providing me a research scholarship and the necessary facilities for the research.

I am highly indebted to my wife Shashi for her patience and understanding and to my sons Pankaj and Niraj for their affectionate rapport throughout the period of this work.

Finally, I thank Mr. J.N. Saini for typing the manuscript neatly and Mr. R.P. Kapoor for the preparation of drawings.

Balbir Kumar

A B S T R A C T

The research work presented in this thesis is connected with new approaches for the design of differentiators for analog as well as digital systems. Efficient results have been achieved by insisting on mathematical exactitude in the investigations.

In analog filters, various configurations of active as well as passive differentiators are available. However, no concrete results are known with respect to optimum, passive and realizable differentiators. An analysis to highlight the intrinsic constraints on realizability and performance of such differentiators has been carried out in this thesis.

Digital differentiators (DD) constitute an important class of digital processors. The DDs characterized by finite impulse response (FIR) are unconditionally stable and can be designed for linear phase responses. The various available design techniques including the most popular one, viz. minimax relative error (MRE) approximation [7], are either inefficient or use time consuming optimization algorithms to compute the weighting coefficients required in the structures. Moreover, these approximations cannot be successfully adopted for DDs required to perform over a limited frequency range. In the present research, attention has been focussed on a different type of designs viz. maximally linear DDs, which

are suitable for operation over a limited frequency range at low, midband and high frequencies. Mathematical formulas for computing the weighting coefficients for the DDs have also been derived.

In numerous applications, one requires a variable frequency/bandwidth DD. No technique to realize such digital differentiators is known. It has been shown, in the present work, that maximally linear DDs can achieve such operation in an elegant manner.

The digital differentiators, the digital Hilbert transformers (DHT) and the half-band low-pass filters (1/2-LPF) apparently seem to be closely related to each other. A composite and complete picture of the inter-relationships between the DDs, DHTs and 1/2-LPFs has been portrayed in this thesis. This enables the differentiator designs to be adopted to the design of the DHT and 1/2-LPF.

C O N T E N T S

	Page
Acknowledgement	... i
Abstract	... ii
Contents	... iv
List of Abbreviations and Symbols	... viii
 CHAPTER-1	
INTRODUCTION	... 1
1.1 Analog differentiators	... 2
1.2 Digital differentiators	... 3
1.2.1 Digital differentiators for low frequencies	... 5
1.2.2 Digital differentiators for midband frequencies	... 7
1.2.3 Digital differentiators for high frequencies	... 7
1.2.4 Variable frequency digital differentiators	... 10
1.2.5 Interrelationships	... 11
1.3 Scope of the work	... 11
1.4 Organization of the thesis	... 12
 CHAPTER-2	
OPTIMUM PASSIVE ANALOG DIFFERENTIATORS	... 15
2.1 Introduction	... 16
2.2 Optimal transfer function and its realizability	... 17
2.3 2nd-order optimal and suboptimal differentiators	... 27
2.4 3rd-order suboptimal passive differentiators	... 30

		Page
2.5	Optimal RC differentiators	... 31
2.6	Suboptimal RC differentiators	... 32
2.7	Conclusions	... 33
2.8	Appendix A2.1	... 36
CHAPTER-3	DESIGN OF DIGITAL DIFFERENTIATORS FOR LOW FREQUENCIES	... 39
3.1	Introduction	... 40
3.2	Derivation of maximally linear DDs from maximally flat LPFs	... 42
3.3	Performance	... 44
3.4	Recursive formula for d_i	... 46
3.5	Explicit formula for d_i	... 50
3.6	Comparison of the three formulas for d_i	... 51
3.7	Conclusions	... 54
3.8	Appendices	... 55
	3.8.1 Appendix A3.1	... 55
	3.8.2 Appendix A3.2	... 58
CHAPTER-4	DESIGN OF DIGITAL DIFFERENTIATORS FOR MIDBAND FREQUENCIES	... 60
4.1	Introduction	... 61
4.2	Derivation of mathematical relation for the weighting coefficients	... 63
4.3	Performance	... 69
4.4	Conclusions	... 75
4.5	Appendix A4.1	... 76

		Page
CHAPTER-5	DESIGN OF DIGITAL DIFFERENTIATORS FOR HIGH FREQUENCIES	... 80
5.1	Introduction	... 81
5.2	Design	... 83
5.3	Realization	... 87
5.4	Performance	... 87
5.5	Conclusion	... 91
CHAPTER-6	VARIABLE FREQUENCY, UNIVERSAL DIGITAL DIFFERENTIATORS	... 92
6.1	Introduction	... 93
6.2	Differentiators for midband frequencies	... 95
6.2.1	Explicit formula for d_i	... 95
6.2.2	Derivation of p_i	... 97
6.3	Differentiators for low frequencies	... 99
6.4	Implementation	... 100
6.5	Performance	... 102
6.6	Conclusion	... 107
CHAPTER-7	INTER-RELATIONS BETWEEN DIGITAL DIFFEREN- TIATORS, HILBERT TRANSFORMERS AND HALF- BAND LOW-PASS FILTERS	... 109
7.1	Introduction	... 110
7.2	Interrelationships	... 111
7.2.1	Relations connecting the impulse responses	... 111
7.2.2	Relations between $H_D(\omega)$ and $H_H(\omega)$	... 113
7.2.3	Relations between $H_H(\omega)$ and $H_L(\omega)$, and between $H_L(\omega)$ and $H_D(\omega)$	... 117

	Page
7.3 Performance	... 120
7.4 Conclusion	... 124
CHAPTER-8 CONCLUSIONS	... 125
8.1 Main results of the thesis	... 126
8.1.1 Analog differentiators	... 126
8.1.2 Digital differentiators	... 126
8.1.3 Interrelations	... 128
8.2 Scope for further research	... 128
R E F E R E N C E S	... 131
PUBLICATIONS/PATENTS BASED ON THE RESULTS OF THIS THESIS	... 136
BIO-DATA	... 138