ROBUST-OPTIMAL CONTROL FRAMEWORK FOR NONLINEAR SYSTEMS

by

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Submitted

in fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

to the

Indian Institute of Technology Delhi

DECEMBER 2009

To

My Wife
Hetal
&
Our Beloved Son
Jwalin

Certificate

This is to certify that the thesis entitled **"Robust-optimal Control framework for Nonlinear Systems"**, being submitted by **Mr. Dipak Kumar Adhyaru** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is a record of bona-fide research work carried out by him under our supervision. In our opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma.

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Acknowledgements

Any piece of work that has proved its way remains incomplete if the sense of gratitude and respect is not being deemed to those who have proved to be supporting during the development period. Though these words are not enough but they can at least pave to understand the feeling of respect and admiration.

My first thanks are to the almighty who bestowed me with his grace and strength all along the work.

It gives me great pleasure to take this opportunity to thank and express my deep sense of gratitude to my supervisors, Prof. I. N. Kar, and Prof. M.Gopal, for their personal involvement in every facet of this work. Their readiness to resolve any point of confusion by mutual discussion, even for hours at a stretch, is really commendable. I deem it my privilege to have carried out this dissertation work under their valuable guidance.

I am indebted to my Student Research Committee members Prof.R.K.P. Bhatt, Prof. Jayadeva and Prof. B.S. Panda for their constructive suggestions and critical comments during my research work.

I would always be grateful to the faculty members and other staff of Electrical Engineering Department, IIT Delhi for their help and support. I would like to specially mention the help received from Mr. Jaipal Singh for their valuable laboratory support and for creating homely and working atmosphere in the laboratory during my tenure.

I can never forget the co-operation, friendly behavior and technical support provided by my colleagues Bharat, Saurabh, Hitesh, Arun, Gopinath, Hasan, Pankaj, Satya, Paramita, Seema, Dhaval and Anurag.

I am thankful to Nirma University Ahmedabad, for sponsoring my PhD at IIT-Delhi. My sincere thanks to all the colleagues of I.C. department of Nirma University who have given more love, respect, encouragement and acceptance than I could have ever hoped for.

On a personal note, my heartfelt gratitude for my father and mother who motivated and supported me throughout this uphill task. Without their blessings this work could not have been accomplished. I have no words for my parents and in-laws as they provided continuous inspiration and spiritual regards throughout my research.

Finally, I thank my wife Hetal whose support, faith and belief in my capability gave me immense inspiration. During our years together here I have relied greatly upon her love, optimism and cheerful disposition to sustain me. Despite the challenges and difficulties, she has never wavered in her devotion. In gratitude, I dedicate this dissertation to her.

Finally, I would like to thank all who were directly or indirectly involved to bring out this work as a successful one.

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Abstract

In optimal control design, we need information about nominal system model. Optimal controllers can be designed by solving the well known Hamilton-Jacobi-Bellman (HJB) equation, but it is very difficult to find an exact solution of it, except for simple problems. Some researchers assume the solution of HJB equation and formulate optimal control problem into a standard linear quadratic regulator problem. Approximate solutions of the HJB equation can be found using the well known techniques like successive Galerkin's approximation method and Neural Network (NN)-based method. NN based optimal control employs a method to approximate a value-function which gives an approximate solution of the HJB equation. The design of control systems requires one to consider various types of constraints and performance measures. Constraints encountered in control systems design are due to physical limitations imposed on the controller and the plant. This includes actuator saturation and constraints on the states. Performance measures on the other hand are related to optimality issues. This includes objectives like minimum fuel, minimum energy, minimum time and robustness. Combining constraints with performance measures requires, in general, solving complicated optimal control problems. Only in limited cases, one may obtain a closed-form solution of the controller. However, the design of a constrained optimal control using the solution of HJB equation is a challenging problem. Attempts have been made in the recent past to find constrained control laws using a non-quadratic performance functional and still the research work is going on. All these HJB-based optimal controller designs need exact information about the system model and nominal model. However, all practical control systems have to be robust with respect to model

uncertainties such as unknown or partially known time-varying process parameters, exogenous disturbances, etc. So, model uncertainties need to be considered during the process of controller design to avoid deterioration of nominal closed-loop performance. In other words, we need to design a robust feedback control law to tackle system uncertainties.

Considerable efforts have been made for the development of robust control theory in the recent years. It has been recognized that the Lyapunov concept and the Hamilton-Jacobi theory are the major analytic paradigms for designing and analyzing uncertain systems. A realistic approach is to design an HJB equation with modified performance functional that can tackle system uncertainties and constraints on the input. An approximate solution of the HJB equation can be obtained using NN. However, constrained robust controller design using NN-based HJB solution has not been explored till date.

The present research puts across a constrained robust controller design for continuous-time and discrete-time nonlinear systems using optimal control approach. The optimal control problem is formulated using modified performance functional to tackle system uncertainties and constraints on the input. The HJB equation-based formulation to solve corresponding optimal control problem is proposed. Neural Networks have been used to find an approximate solution of the HJB equation with modified performance functional. The proposed algorithm has been applied on various classes of systems like matched and unmatched uncertain systems, weakly coupled nonlinear systems and discrete-time systems. With theoretical proofs, it has been shown that the Lyapunov function guaranteeing stability is the solution of HJB equation of the nominal system.

Contents