

**PERFORMANCE OF OPTICAL SYSTEMS IN PRESENCE  
OF IMAGE MOTION**

**BY**

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**THESIS SUBMITTED TO THE INDIAN INSTITUTE OF  
TECHNOLOGY, DELHI**

**FOR THE AWARD OF THE DEGREE OF  
DOCTOR OF PHILOSOPHY**

**DEPARTMENT OF PHYSICS  
INDIAN INSTITUTE OF TECHNOLOGY  
DELHI**

**JULY 1975**

**DEDICATED TO**  
**MY RESPECTABLE PARENTS**

## ACKNOWLEDGEMENTS

I wish to express my deep sense of gratitude to Prof. C.L. Mehta and Mr. Kehar Singh for their guidance, patience, magnanimity, stimulating counsel and ever helpful attitude during all phases of this work.

I wish to thank Prof. M.S. Sodha and A.K. Ghatak for their constant interest in the work. Thanks are also due to my colleague Mr. P.C. Gupta for his constant help and friendly co-operation. My thanks are also due to optics research group and computer centre staff for their help and co-operation.

My special thanks are also due to Dr. George Joseph of the Space Applications Centre, Ahmedabad who consistently encouraged and consoled me in my efforts and often turned despair into renewed hope.

I wish to express my special appreciation to Dr. T.A. Hariharan, Mr. D.S. Kamat, Mr. A.K.S. Gopalan, Mr. H.S. Raina, Mr. U.G. Desai, Mr. N.K. Vyas and Mr. N.V. Maslekar of the Remote Sensing and Meteorology Applications Division, Space Applications Centre for their encouragement.

I am deeply indebted to my respectable parents, brothers and all my relatives who had been a constant source of inspiration and had been very helpful in cultivating an interest for learning. The financial support from the Council of Scientific and Industrial Research, India is gratefully acknowledged.

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## PREFACE

Theoretical and experimental analysis of intensity distribution in the image of sinusoidal gratings of different spatial frequencies is now well known. The intensity distribution in the images can be obtained by convolution of the object function with the appropriate spread function of the optical system.

Investigations on the images of infinite periodic sine wave objects are fundamental to the concept of optical transfer function ( O.T.F. ). Investigations on the images of objects with finite number of cycles have recently assumed importance in connection with the accuracy of O.T.F. measurement and in aerial photography, where we may have objects which are periodic in nature. The study of isolated targets is also important due to the fact that the information on the performance for isolated compact details is quite often required. It is strongly felt that the important information in aerial photographs is frequently conveyed by lines or edges, and the resolving power, as determined by compact targets, possesses obvious apparent advantages in this respect.

Image formation in presence of aberrations and constructional defects has been studied extensively. However, the effects of transverse and  $\odot$  longitudinal vibrations, or the linear and parabolic image motions have not been considered adequately. In spite of image motion

compensation techniques that are being used, there is always some residual motion present. These problems have become more important with the advancement of technology in the field of aircraft, which have gained considerable height and speed. In many situations one needs to know the image motion accurately. Keeping these points in view, we have investigated the effect of image motion on the images of various objects of both periodic and aperiodic type. Results are likely to have applications in setting of tolerances on image motion, stabilization, image motion compensation, motion analysis using diffraction techniques and study of coherence via diffraction experiments etc.]

It is proposed to divide the thesis in five chapters and three appendices on related topics.]

In the first chapter an introduction to the subject of image formation is given. It is then shown how and where the image motion occurs. The available results for O.T.F. are summarized for the cases of transverse and longitudinal sinusoidal vibrations and linear and parabolic image motions.]

In the second chapter we have discussed diffraction images of truncated square and sine wave objects in presence of image motion. Fourier transform techniques have been used to evaluate the image intensity distribution. It has been found that the number of cycles in the target required to approximate a target with infinite number of periods in the

presence of image motion is more than that in the absence of motion. The exact number depends upon the amount of image motion. The loss in periodicity and spurious resolution in the image have been investigated in detail. At the end of this chapter we have investigated the images of infinite rectangular and triangular objects in the presence of parabolic image motion which occurs in panoramic aerial cameras.

In the third chapter we have considered the effect of image motion on the images of isolated objects such as straight edges or single cycles of triangular, sine and square objects. Since periodic test charts are not usually available in the image format, alternative techniques such as edge gradient analysis are frequently employed. M.T.F. evaluation using edge trace analysis has for example been used to assess the image motion compensation capabilities of the lunar orbiter photographic system in actual operation and preflight laboratory testing. Detailed results have been given for the variation of edge gradient, isophotes and relief images. In presence of parabolic motion, particular note should be taken of the position of the edge. We have also discussed the motion degraded bar spread function and results are compared with the images of targets in the form of single cycle of triangular and sinusoidal objects.

In the fourth chapter the effect of longitudinal and transverse sinusoidal vibrations on the diffraction images of bright incoherent objects is considered. An expression for  $I(V, \psi)$ , the intensity distribution in the

image plane is derived for circularly symmetric objects and is given by,

$$I(V, \psi) = \frac{1}{2\pi} \int_0^2 \omega d\omega \int_0^{2\pi} C(\omega) O(\omega) J_0(\pi A \omega \cos \psi \cos(\theta - \psi)) \times \cos(V \omega \cos(\theta - \psi)) d\theta$$

Here the various symbols have the following meanings

- $C(\omega)$  - Transfer function of diffracting system;  
 $O(\omega)$  - Object spatial frequency spectrum;  
 $A$  - Vibration parameter in dimensionless units  
 $V$  - Distance in the image plane in reduced units;  
 $\psi$  - Angle contained between the direction of vibration and direction of observation in the image plane;  
 $J_0$  - Bessel function of order zero.

The results thus obtained have also been verified (for small disk diameters) by evaluating the time average intensity of a vibrating Airy disk. It is seen that in certain cases the influence of vibrations is very pronounced and may completely distort the object. For example for some values of the vibration parameter a disk may be interpreted as an annulus and vice versa. The intensity distributions have been calculated in the image plane for  $\psi = 0^\circ$  and  $45^\circ$ .

In the last chapter we consider the effect of partial coherence and image motion on the diffraction images. The analysis makes use of a suitably modified form of shells

theorem. The characteristics of the point spread function have been obtained for several values of the degree of coherence and motion parameters. Four forms of coherence functions have been used namely,  $\text{besinc}$ ,  $\text{sinc}$ , exponential and gaussian. It may be inferred from these results that the effect of image motion becomes less pronounced in case of partially coherent illumination. It is also shown that for some values of the degree of coherence and motion parameter the splitting of the main lobe occurs which gives rise to spurious resolution. In the end we compare the results derived in this chapter with those of chapter four.

In appendix A we use Hopkins' formulation for the imagery in partially coherent illumination to investigate the effect of amplitude filter on the diffraction images of bar targets. The decrease of ringing in the image with amplitude filter and degree of coherence has been investigated.

In appendix B the effect of astigmatism on the diffraction images of edge objects has been considered when the image plane is midway between the tangential and meridian foci. A detailed analysis has been made of the effect of decrease in the edge gradient with increasing amount of aberration. This decrease is rapid for small amounts of aberrations than for large values.

In appendix C an expression is derived for the intensity distribution in the fringes of equal inclination

formed by a Fabry perot interferometer in the presence of transverse sinusoidal vibrations of one of the plates having surface imperfections. The resulting errors in the measurement of vibration amplitude have been discussed. A part of the work reported in this thesis has resulted in the following research articles:

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