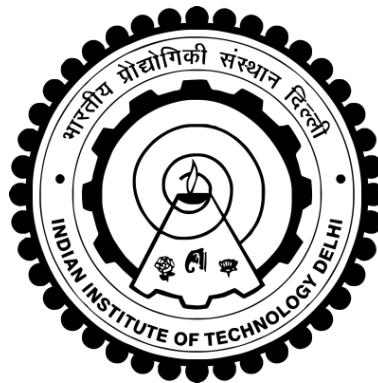


**CONTROL OF UNCERTAIN NONLINEAR SYSTEMS WITH
UNKNOWN TIME DELAY**

ASHISH KUMAR JAIN



**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

FEBRUARY 2020

© Indian Institute of Technology Delhi (IITD), New Delhi, 2020

**CONTROL OF UNCERTAIN NONLINEAR SYSTEMS WITH
UNKNOWN TIME DELAY**

by

ASHISH KUMAR JAIN

Department Of Electrical Engineering

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

FEBRUARY 2020

CERTIFICATE

This is to certify that the thesis entitled **Control of Uncertain Nonlinear Systems with Unknown Time Delay** submitted by **Ashish Kumar Jain** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Prof. Shubhendu Bhasin
Department of Electrical Engineering,
Indian Institute of Technology Delhi.
(Supervisor)

ACKNOWLEDGEMENTS

First of all, I would like to express my deepest gratitude to the Almighty for obliging me with the strength for completing this research successfully. I am using this opportunity to express my gratitude to everyone who supported me throughout the course of this thesis.

I am profoundly grateful to my supervisor Dr. Shubhendu Bhasin for his aspiring guidance, invaluable suggestions and continuous encouragement over the years during the period of my PhD study here at IIT Delhi. My research would have been impossible without the support of his patience, motivation and immense knowledge.

My heartfelt thanks goes to my thesis committee members: Prof. I. N. Kar, Dr. S. Janardhanan and Prof. S. K. Saha, not only for their insightful feedback and encouragement, but also for the constructive criticism which incentivized me to widen my research from various perspectives. I would also want to express my regards to Prof. I. N. Kar, Dr. S. Janardhanan, Dr. S. Bhasin and Dr. Shaunak Sen for their teachings which broadened my insights of various concepts in control systems. I would always be thankful to the staff members of Control and Automation group, Electrical Engineering Department of IIT Delhi, especially to Virender, for their unfailing support and assistance.

I would also like to thank my colleagues and dear friends Koena, Madan, Satnesh, Sumit, Gagan, Niraj, Sayan, Atul, Nalin, Joyjit for their thorough support.

I acknowledge that this dissertation would not have been possible without the financial support provided by Mahatma Jyotiba Phule Rohilkhand University, Bareilly, Uttar Pradesh. Last but not the least, I would like to thank my family: parents, brother, sister and wife Rachana for their unconditional support and constant encouragement throughout this wonderful journey called PhD.

Ashish Kumar Jain

ABSTRACT

The problem of stabilization of systems with input delay has been well researched in the last few decades, however, challenges still exist in addressing the problem, owing to lack of information about the delay, uncertainty in the system dynamics and external disturbances. Since delay in the input often leads to instability, undesired oscillations in system's response and poor performance, it is required to compensate for effect of the delay. A typical time-delay control design requires knowledge of delay and complete system information to provide acceptable tracking results. On the other hand, robust and adaptive control methodologies can be used for compensation of time delay in uncertain systems by estimating/approximating the system dynamics and delay or delay bound.

The focus of this thesis is to investigate the problem of control design for nonlinear systems with unknown input time-delay, uncertain dynamics and external disturbances. This research intends to address some of the aforementioned limitations by proposing a novel compensator design, wherein a filtered tracking error signal $r(t)$ is introduced using the finite integral of previous values of control signal $u(t)$ where the limits of integration are independent of the delay. The sufficient inequality conditions on controller gains and upper bound of input delay are derived in stability analysis by choosing suitable L-K functional, which guarantee a global uniformly ultimately bounded (GUUB) tracking result.

The control law consists of three terms, where first term is a standard PI (Proportional - Integral) controller where the integral action provides robustness to external disturbances including time delays as proposed in [1]. The second term corresponds to the delay compensation term as it includes a finite integral of the past control values in the interval $\left[t - \bar{\tau}_i \quad t \right]$. This term is responsible for canceling the input delay term $u(t - \bar{\tau}_i)$ in the stability analysis. The last term is used purely for stability reasons,

specifically to cancel the leftover positive terms from the LK functional derivative, in the stability analysis. Overall, the controller is composed of a PI controller + delay compensator (based on a finite integral of the past control values).

Further, this design is extended for the problem of a class of nonlinear dynamical systems subjected to parametric uncertainty and simultaneous unknown, time-varying state and input delays.

सारांश

इनपुट में देरी के साथ सिस्टम के स्टेबिलिटी की समस्या पर पिछले कुछ दशकों में अच्छी तरह से शोध किया गया है। हालांकि ऐसे सिस्टम जिनकी डायनामिक्स और इनपुट में देरी की अनिश्चितता के बारे में जानकारी का अभाव हो, उनकी नियंत्रण की समस्या अभी भी चुनौतिपूर्ण हैं। चूंकि इनपुट डिले प्रायः इनस्टेबिलिटी और सिस्टम की रेस्पॉन्सेस में उपेक्षित ओसिलेशंस को बढ़ता है और आउटपुट को खराब करता है, तब डिले के प्रभाव को समाप्त करना आवश्यक हो जाता है। अपेक्षित ट्रैकिंग परिणाम के लिए एक विशिष्ट कण्ट्रोल डिज़ाइन में डिले एवं पूर्ण तंत्र की सुचना आवश्यक है। दूसरी और अनिश्चित सिस्टम्स में डिले के कंपेन्सेशन के लिए रोबस्ट और एडप्टिव नियंत्रण का उपयोग किया जा सकता है।

ऐसे तंत्र जिनकी अनिश्चित गतिकी, डिले और डिस्टर्बेंसेस जिनकी सुचना अपूर्ण हो, उनके नियंत्रण की प्रोब्लम्स का समाधान इस शोध में किया गया है। यह शोध एक नवीन कण्ट्रोल डिज़ाइन को प्रस्तावित करता है जिसमें एक फिल्टर्ड ट्रैकिंग एरर सिग्नल $r(t)$ और कण्ट्रोल इनपुट $u(t)$ की पिछली वैल्यूज को उपयोग करते हैं, और इंटीग्रेशन की लिमिट डिले पर निर्भर नहीं होंगी। अनुकूल L-K फंक्शनलस को चुनते हुए स्टेबिलिटी एनालिसिस में कंट्रोलर गेन्स और इनपुट डिले की अपर बाउंड की पर्याप्त इनइक्लिटी कंडीशंस को निकला गया है जो की ग्लोबल युनिफॉर्मली अलटिमटेली बॉण्डेड ट्रैकिंग परिणाम होने की गारंटी प्रदान करती है।

नियंत्रण नियम की समीकरण में तीन पद हैं। जिसमें प्रथम पद P-I कंट्रोलर है जहाँ इंटीग्रल एक्शन बाहरी डिस्टर्बेंस पदों के प्रति रोबस्टनेस प्रदान करता है। द्वितीय पद डिले कंपेन्सेशन पद है, जिसमें कण्ट्रोल इनपुट के पिछले पदों का फाईनाइट इंटीग्रेशन सम्मिलित है। यह पद स्टेबिलिटी एनालिसिस में इनपुट डिले पद को समाप्त करने में उपयोगी है। अंतिम पद, जो की धनात्मक पद रह जाते हैं, उनको समाप्त करने में उपयोगी है। संक्षिप्त में कंट्रोलर, एक P-I कंट्रोलर पद एवं डिले कम्पेन्सेटर पदों से बना है, कहा जा सकता है।

आगे, नॉनलीनियर सिस्टम्स जिनमें स्टेट और इनपुट डिले दोनों मौजूद है, उनकी समस्या के समाधान के लिए भी इस डिज़ाइन को विस्तारित किया गया है।

Contents

1	Introduction	1
1.1	Preliminaries of Time Delay Systems	1
1.1.1	Systems with Time Delay	1
1.1.2	Classification of Functional Differential Equation (FDE)	3
1.1.2.1	Retarded Functional Differential Equation (RFDE)	3
1.1.2.2	Neutral Functional Differential Equation (NFDE)	3
1.1.3	Transfer Function of Delay	4
1.1.4	Stability Analysis of Time Delay Systems	4
1.1.4.1	Lyapunov-Krasovskii Functional	5
1.1.4.2	Razumikhin Function Approach	6
1.1.4.3	Cluster Treatment of Characteristic Roots (CTCR)	6
1.2	Motivation and Literature Survey	9
1.3	Contribution of the Thesis	14
1.4	Organization of the Thesis	15
2	Overview of Control Design Techniques for Input Delay Systems	18
2.1	Control Design for Linear Systems with Input Delay	18
2.1.1	Smith Predictor Approach	19
2.1.2	Artstein Model Reduction	20

2.1.3	Predictor Feedback Design for ODE with Actuator Delay	21
2.1.4	Backstepping Transformation	22
2.1.5	Control Design for Unknown, Constant Delay	23
2.1.6	Control Design for Known, Time-Varying Delay	23
2.2	Control design for Nonlinear systems with input delay	24
2.2.1	Predictor Feedback Design	24
2.2.2	Control Design for Euler-Lagrange System with Known Constant Delay	25
2.3	Conclusions	26
3	Tracking Control of Uncertain Nonlinear Systems with Unknown Con- stant Input Delay	28
3.1	Introduction	28
3.2	Problem Formulation and Assumptions	29
3.3	Control Development	31
3.4	Stability Analysis	33
3.5	Simulation	39
3.6	Conclusions	41
4	Unknown Time-Varying Input Delay Compensation for Uncertain Non- linear Systems	42
4.1	Introduction	42
4.2	System Dynamics	43
4.3	Control Development	44
4.4	Stability Analysis	46
4.5	Simulation	52
4.5.1	Example 1	52

4.5.2	Example 2	54
4.6	Extension to Euler-Lagrange Systems	57
4.6.1	System Dynamics Properties & Assumptions	57
4.6.2	Simulation	58
4.7	Conclusion	60
5	Simultaneous Unknown Time-Varying State and Input Delays Compensator Design for Uncertain Nonlinear Systems	62
5.1	Introduction	62
5.2	Problem Formulation and Assumptions	63
5.3	Control Law Design	65
5.4	Stability Analysis	67
5.5	Simulation	73
5.5.1	Example 1	73
5.5.2	Example 2	75
5.5.3	Example 3	77
5.6	Conclusions	78
6	Conclusion and Future Directions of Research	80
6.1	Conclusion	80
6.2	Future Work	81
A	Proofs	83
A.1	Proof of Lemma 1 in Chapter 3	83
A.2	Proof of Lemma 2 in Chapter 4	84
A.3	Proof of Lemma 3 in Chapter 5	86

List of Figures

1.1.1 Block diagram of delay model [2].	2
2.1.1 Smith predictor [3].	19
3.5.1 Tracking error and control input response for the delay bound of 1 sec.	40
3.5.2 Tracking error and control input response for different values of variable delay and the delay bound of 1 sec.	41
4.5.1 Tracking error, control input response and delay for Case 1 and 2.	53
4.5.2 Tracking error response for Case 1 and 3.	53
4.5.3 Tracking error, control input response and delay for sinusoidal delay with delay bound of 1 sec. for Example 2.	55
4.5.4 Tracking error, control input response and delay for random delay with delay bound of 1 sec. for Example 2.	56
4.6.1 Tracking errors, control input response and delay for Case 1.	60
4.6.2 Tracking errors, control input response and delay for Case 3.	60
5.5.1 Tracking error and control input response for the Case 1.	74
5.5.2 Tracking error and control input responses for the Case 2.	75
5.5.3 Tracking error and control input responses for the Case 3.	75
5.5.4 Tracking error and control input response for the Example 2.	76

5.5.5 Comparison of states and control law responses obtained from proposed method and Jiao et. al. [4]	78
---	----

List of Tables

3.5.1 Tracking errors for different values of delay and a fixed delay bound of 1 sec.	39
4.5.1 Tracking error and control gains for different sinusoidal delays for Example 1	54
4.5.2 Tracking errors and control gains for different values of sinusoidal delay and a fixed delay bound of 1 sec. for Example 2	55
4.6.1 Tracking errors for the delay bound of 2 sec.	59
5.5.1 Tracking errors for different types of state and input delays	74

List of Abbreviations

LK	Laypunov-Krasovskii
E-L	Euler-Lagrange
GUUB	Global Uniformly Ultimately Boundedness
RMS	Root Mean Square
DCAL	Desired Compensation Adaptation Law

List of Symbols

\mathbb{R}	Real Space
\mathbb{R}^+	Positive real value
\mathbb{R}^n	Real Vector Space of dimension n
$\mathbb{R}^{n \times m}$	Real Matrix Space of dimension $n \times m$
I	Identity Matrix with appropriate dimension
\mathcal{L}_∞	The space of bounded Lebesgue measurable functions on $[0, \infty)$
$\ \cdot\ $	Standard Euclidean norm