MULTI-HAZARD STOCHASTIC RESPONSE CONTROL OF STRUCTURES USING SMART BASE ISOLATION SYSTEMS

DANIEL HABTAMU ZELLEKE

DEPARTMENT OF CIVIL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY (IIT) DELHI

HAUZ KHAS, NEW DELHI - 110 016, INDIA

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by

DANIEL HABTAMU ZELLEKE

DEPARTMENT OF CIVIL ENGINEERING

submitted

in fulfilment of the requirements for the award of the degree of

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DEDICATION

Dedicated to my beloved parents and all teachers who have devoted their lives to sharing knowledge.

CERTIFICATE

This is to certify that the PhD Thesis titled "**MULTI-HAZARD STOCHASTIC RESPONSE CONTROL OF STRUCTURES USING SMART BASE ISOLATION SYSTEMS**" being submitted by **DANIEL HABTAMU ZELLEKE** to the Indian Institute of Technology (IIT) Delhi for the award of the degree of **DOCTOR OF PHILOSOPHY** is a record of the bonafide research work carried out by him. He worked under my supervision for the submission of this PhD Thesis, which to my knowledge has reached the requisite standard as demonstrated by excellent publications in international journals and conferences.

Further, the contents of his research work, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma to the best of my knowledge and belief.

New Delhi,
India.

Prof. Dr Vasant A. MATSAGAR
Professor, Dogra Chair, and Head,
Department of Civil Engineering,
Indian Institute of Technology (IIT) Delhi,
Hauz Khas, New Delhi - 110 016, India.

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ABSTRACT

Ensuring safety and post-hazard functionality of structures under dynamic loads, such as earthquakes, wind, and blasts, are some of the significant challenges in the current rapidly growing urban environment. Various structural response control techniques have been used for response control of structures under different hazards. Base isolation is a popular response control strategy that has been implemented for effective earthquake protection of structures. Although base-isolated structures are also likely to be subjected to non-seismic hazards, limited attention is given to the application of base isolation for response mitigation under multi-hazard scenarios. Particularly, more research is needed on the application of smart base isolation (SBI) strategies for response control under various hazards. Therefore, this doctoral research aims to investigate the application of SBI systems for response mitigation of structures under independent multi-hazard scenarios.

First, the dynamic response of structures equipped with passive base isolation systems is numerically studied under earthquake ground motions (EQGMs), wind load (WL), and blast-induced ground motions (BIGMs). Here, deterministic and stochastic numerical investigations are performed to study the effects of various parameters of isolation systems and superstructure properties on the behavior of base-isolated buildings under the three types of hazards. Also, the polynomial regression-based Monte Carlo simulation (PRBMCS) method is newly proposed for the fragility assessment of the base-isolated buildings. The results of the deterministic and stochastic assessments highlight the need for careful consideration of all expected hazards in designing base isolators.

Subsequently, a novel semi-active control algorithm, i.e., the energy-based predictive (EBP) algorithm, is developed and numerically evaluated for the suppression of undesirable structural vibrations under dynamic loads. In this algorithm, the mechanical energy of the vibrating structure is considered the primary variable influencing the control action. The proposed strategy is implemented for response control of structures using semi-active tuned mass dampers (SATMDs), and its efficacy is numerically studied under sinusoidal excitation, EQGM, and WL. Moreover, the influence of the SATMD parameters on control performance is assessed, based on which empirical relations are proposed to identify SATMD parameters suitable for optimum control performance.

The application of a semi-active hybrid base isolation strategy, consisting of an EBP algorithm-controlled SATMD, is investigated numerically for mitigation of structural response under EQGM, WL, and BIGM. Three configurations of the SBI strategy are presented, and modifications to the EBP algorithm are proposed for application to base-isolated structures equipped with the SATMDs. Also, deterministic and stochastic studies are performed to assess the suitability of the SBI strategies for vibration control of structures under EQGM, WL, and BIGM. Importantly, the merits and limitations of the SBI strategy in response mitigation under the three types of hazards are highlighted.

A magnetorheological (MR) damper-based SBI strategy controlled with the EBP algorithm is proposed. Here, the EBP algorithm is adapted for application to MR damper-fitted base-isolated structures and is used to modulate the suitable control signal, i.e., the voltage supply to the MR damper. An experimental investigation is conducted to evaluate the practicability of implementing the EBP algorithm-controlled MR damper for response control of base-isolated structures under seismic action. Accordingly, shake table testing is performed on a single-story base-isolated structure equipped with an MR damper. The results show good agreement between the experimentally measured and numerically quantified response quantities for the passive and semi-active cases, hence validating the practicability of the strategy. Moreover, further experimental and numerical studies are performed to study the implementation of the MR damper-based SBI strategy for response mitigation under BIGM. The applicability of the strategy for response mitigation under BIGM is verified through testing conducted on a two-story base-isolated structure. Also, the performance and limitations of the EBP algorithm in the response mitigation of MR damper-fitted base-isolated structures under EQGM and BIGM are presented.

Finally, a reliability-based multi-hazard design optimization framework is newly proposed for base-isolated buildings. A modified weighted composite desirability function is presented to account for the contradictory design objectives under different hazards. Also, a method of calculating the weights of the individual desirability functions is introduced. The application of the proposed framework for reliability-based optimization of multi-story buildings equipped with elastomeric rubber bearings is studied under multi-hazard scenarios involving EQGM, WL, and BIGM. The findings demonstrate the suitability of the proposed framework for the reliability-based multi-hazard design optimization of base-isolated buildings.

सार

वर्तमान में तेजी से बढ़ते शहरी वातावरण में भूकंप, हवा और विस्फोट जैसे गतिशील बल के तहत संरचनाओं की सुरक्षा और खतरे के बाद की कार्यक्षमता सुनिश्चित करना, कुछ महत्वपूर्ण चुनौतियां हैं। अलग अलग खतरों के तहत संरचनाओं के प्रतिक्रिया नियंत्रण के लिए विभिन्न संरचनात्मक प्रतिक्रिया नियंत्रण तकनीकों का उपयोग किया गया है। स्थल-विभक्त प्रणाली एक लोकप्रिय प्रतिक्रिया नियंत्रण कार्यनीति है जिसे संरचनाओं की प्रभावी भूकंप सुरक्षा के लिए लागू किया गया है। हालाँकि स्थल-विभक्त संरचनाओं के भी गैर-भूकंपीय खतरो के अधीन होने की संभावना है, लेकिन बहु-खतरे परिदृश्यों के तहत संरचनाओं के प्रतिक्रिया शमन के लिए स्थल-विभक्त के अनुप्रयोग पर सीमित ध्यान दिया जाता है। विशेष रूप से, विभिन्न खतरों के तहत प्रतिक्रिया नियंत्रण के लिए प्रज्ञ स्थल-विभक्त (प्रस्थवि) प्रणाली कार्यनीतियों के अनुप्रयोग पर अधिक शोध की आवश्यकता है। इसलिए, इस वाचस्पति अनुसंधान का उद्देश्य स्वतंत्र बहु-खतरे परिदृश्यों के तहत संरचनाओं की प्रतिक्रिया शमन के लिए प्रस्थवि प्रणाली के अनुप्रयोग की जांच करना है।

सर्वप्रथम निष्क्रिय स्थल-विभक्त प्रणालियों से योजित संरचनाओं की गतिशील प्रतिक्रिया का भूकंपकिय ज़मीनी गति (भूजग), पवन बल (पब), और विस्फोट-जनित ज़मीनी गति (विजजग) के तहत अभिकलनात्मक रूप से अध्ययन किया जाता है। यहां तीन प्रकार के खतरों के तहत स्थल-विभक्त इमारतों के व्यवहार पर विभक्त प्रणालियों और अधिरचना विशेषताओ के विभिन्न मापदंडों के प्रभावों का अध्ययन करने के लिए नियतात्मक और संभाव्य अभिकलनात्मक जांच की गई है। इसके अलावा स्थल-विभक्त इमारतों की क्षयिता के आकलन के लिए बहुपद प्रतिगमन-आधारित मोंटे कार्लो अनुकरण (बप्रमोंकाअ) विधि नई प्रस्तावित है। निश्चयात्मक और संभाव्य आकलन के परिणामो द्वारा स्थल-विभक्तर की रचना में सभी अपेक्षित खतरो पर सावधानीपूर्वक विचार करने की आवश्यकता पर प्रकाश डाला गया हैं।

इसके बाद, एक नया अर्ध-सक्रिय नियंत्रण कलन विधि, यानी, ऊर्जा-आधारित अनुमानकर्ता (ऊआअ) कलन विधि विकसित की गई है और गतिशील बल के तहत अवांछित संरचनात्मक कंपन के दमन के लिए अभिकलनात्मक रूप से आकलन किया गया है। इस कलन विधि में, कंपित संरचना की यांत्रिक ऊर्जा को संरचना की नियंत्रण क्रिया को प्रभावित करने वाला प्राथमिक चर माना जाता है। प्रस्तावित कार्यनीति अर्ध-सक्रिय अनुरूपित द्रव्यमान अवशोषक (अद्रअ) का उपयोग करके संरचनाओं के प्रतिक्रिया नियंत्रण के लिए कार्यान्वित किया गया है, और इसकी प्रभावकारिता का साइनसॉइडल उत्तेजना, भूज़ग और पब के तहत अभिकलनात्मक रूप से अध्ययन किया गया है। इसके अलावा, नियंत्रण प्रदर्शन पर अद्रअ मापदंडों के प्रभाव का आकलन किया गया है, जिसके स्थल पर सर्वोत्तम नियंत्रण प्रदर्शन के लिए उपयुक्त अद्रअ मापदंडों की पहचान करने के लिए प्रयोगसिद्ध संबंधों का प्रस्ताव किया गया है।

भूजग, पब और विजजग के तहत संरचनात्मक प्रतिक्रिया को कम करने के लिए ऊआअ कार्यप्रणाली-नियंत्रित अद्रअ से युक्त अर्ध-सक्रिय संकर स्थल-विभक्तप्रणाली कार्यनीति के अनुप्रयोग की अभिकलनात्मक रूप से जांच की गई है। प्रस्थवि कार्यनीति के तीन विन्यास प्रस्तुत किए गए हैं, और अद्रअ से योजित स्थल-विभक्त संरचनाओं के अनुप्रयोग के लिए ऊआअ कार्यप्रणाली में संशोधन प्रस्तावित किया गया हैं। इसके अलावा, भूजग, पब और विजजग के तहत संरचनाओं के कंपन नियंत्रण के लिए प्रस्थवि कार्यनीतियों की उपयुक्तता का आकलन करने के लिए निश्चयात्मक और संभाव्य अध्ययन किए जाते हैं। महत्वपूर्ण रूप से, तीन प्रकार के खतरो के तहत प्रतिक्रिया शमन में प्रस्थवि कार्यनीति की खूबियों और सीमाओं पर प्रकाश डाला गया है।

ऊआअ कलन विधि से नियंत्रित एक चुंबकत्वप्रवाहिकी (चुंप्र) अवशोषक-आधारित प्रस्थवि कार्यनीति प्रस्तावित की गई है। यहां ऊआअ कार्यप्रणाली को चुंप्र अवशोषक-युक्त स्थल-विभक्त संरचनाओं के अनुप्रयोग के लिए अनुकूलित किया गया है और इसका उपयोग उपयुक्त नियंत्रक संकेत, यानी चुंप्र अवशोषक को वोल्टेज आपूर्ति को नियंत्रण करने के लिए किया गया है। भूकंपीय कार्रवाई के तहत स्थल-विभक्त संरचनाओं के प्रतिक्रिया नियंत्रण के लिए ऊआअ कार्यप्रणाली-नियंत्रित चुंप्र अवशोषक को लागू करने की व्यावहारिकता का आकलन करने के लिए एक प्रयोगात्मक जांच की जाती है। तदनुसार, कंपन मेज़ चुंप्र अवशोषक से योजित एकल-मंजिला स्थल-विभक्त संरचना पर परीक्षण किया गया है। परिणाम निष्क्रिय और अर्ध-सक्रिय मामलों के लिए प्रयोगात्मक रूप से और अभिकलनात्मक रूप से अनुमानित प्रतिक्रिओं के बीच अच्छा मिलान दिखाते हैं, इसलिए कार्यनीति की व्यावहारिकता को साबित करते हैं। इसके अलावा, विजजग के तहत प्रतिक्रिया शमन के लिए चुंप्र अवशोषक-आधारित प्रस्थवि कार्यनीति के कार्यान्वयन का अध्ययन करने के लिए आगे प्रयोगात्मक और अभिकलनात्मक अध्ययन किया गया हैं। विजजग के तहत प्रतिक्रिया शमन के लिए कार्यनीति की प्रयोज्यता को दो मंजिला स्थल-विभक्त संरचना पर किए गए परीक्षण के माध्यम से सत्यापित किया गया है। इसके अलावा, भूजग और विजजग के तहत चुंप्र अवशोषक-युक्त स्थल-विभक्त संरचनाओं की प्रतिक्रिया शमन में ऊआअ कार्यप्रणाली का प्रदर्शन और सीमाएं प्रस्तुत की गई हैं।

अंत में, स्थल-विभक्त इमारतों के लिए एक विश्वसनीयता-आधारित बहु-खतरा रचना अनुकूलन रूपरेखा प्रस्तावित किया गया है। विभिन्न खतरो के तहत विरोधाभासी रचना उद्देश्यों को ध्यान में रखते हुए एक संशोधित भारित समग्र वांछनीयता फलन प्रस्तुत किया गया है। साथ ही, विशिष्ट वांछनीयता फलन के भार की गणना करने की एक विधि भी पेश की गई है। प्रत्यास्थलिक रबर बियरिंग्स से योजित बहुमंजिला इमारतों के विश्वसनीयता-आधारित अनुकूलन के लिए प्रस्तावित रूपरेखा के अनुप्रयोग का अध्ययन भूजग, पब और विजजग से जुड़े बहु-खतरे परिदृश्यों के तहत किया गया है। निष्कर्ष स्थल-विभक्त इमारतों की विश्वसनीयता-आधारित बहु-खतरा रचना अनुकूलन के लिए प्रस्तावित रूपरेखा की उपयुक्तता को प्रदर्शित करते हैं।

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ABBREVIATIONS

2CC	Second-cost-cumulant
2D	Two-dimensional
3D	Three-dimensional
ATMD	Active tuned mass damper
BI	Base-isolated
BIGM	Blast-induced ground motion
BI-PassiveOFF	Base-isolated structure with magnetorheological damper passively controlled using a constant supply of the minimum voltage value
BI-PassiveON	Base-isolated structure with magnetorheological damper passively controlled using a constant supply of the maximum voltage value
BI-PTMD	Base-isolated building with passive tuned mass damper
BI-SATMD	Base-isolated building with semi-active tuned mass damper
BI-SmartMR	Base-isolated structure with semi-actively modulated magnetorheological damper
BLQG	Balanced linear quadratic Gaussian
CCD	Central composite design
CMS	Conditional mean spectrum
COV	Coefficient of variation
DAQ	Data acquisition
DFA	Desirability function analysis
dM-PTMDs	Distributed multiple passive tuned mass dampers
dM-SATMDs	Distributed multiple semi-active tuned mass dampers
DOE	Design of experiment
DVA	Dynamic vibration absorber
EBCS	Ethiopian building code standard
EBP	Energy-based predictive
E-PB	Energy-to-peak based
EQGM	Earthquake ground motion
FB	Fixed-base
FBR	Fixed-base response
FC	Fuzzy controller
FF	Far-fault

FFT	Fast Fourier transform
FPS	Friction pendulum system
FRF	Frequency response function
GLS	Global least squares
GUI	Graphical user interface
HDRB	High damping rubber bearing
IIT	Indian institute of technology
LCRB	Lead-core rubber bearing
LQG	Linear quadratic Gaussian
LQG-α	Linear quadratic Gaussian-alpha
LQR	Linear quadratic regulator
LRB	Laminated rubber bearing
MaTB	Maximum-the-best
MCE	Maximum considered earthquake
MCS	Monte Carlo simulation
MDOF	Multi-degree of freedom
MHPS	Multi-hazard protective structures
MiTB	Minimum-the-best
ML	Machine learning
MLS	Moving least squares
MLS-RSM	Moving least squares-based response surface methodology
MPC	Model predictive control
M-PTMDs	Multiple passive tuned mass dampers
MR	Magnetorheological
MR-STMD	Tuned mass damper with magnetorheological damper
M-SATMDs	Multiple semi-active tuned mass dampers
MSMC	Modified sliding mode control
MTMD	Multiple tuned mass damper
NF	Near-fault
NR	Northridge
PDF	Probability density function
PEER	Pacific Earthquake Engineering Research Centre
PF system	Pure friction system
PGA	Peak ground acceleration

PID	Proportional-integral-differential
P-PB	Peak-to-peak based
PRBMCS	Polynomial regression-based Monte Carlo simulation
PSDF	Power spectral density function
PTMD	Passive tuned mass damper
RC	Reinforced concrete
R-FBI	Resilient-friction base isolator
RMS	Root mean square
RSM	Response surface methodology
SAIVS-TMD	Semi-active variable stiffness tuned mass damper
SATMD	Semi-active tuned mass damper
SBI	Smart base isolation
SDOF	Single-degree of freedom
SMFC	Sliding mode fuzzy control
S-PTMD	Single passive tuned mass damper
SRS	Sinusoidal reference strategy
S-SATMD	Single semi-active tuned mass damper
SSI	Soil-structure interaction
TCFP	Triple concave friction pendulum
TFP	Triple friction pendulum
TMD	Tuned mass damper
TNT	Trinitrotoluene
TTB	Target-the-best
UHS	Uniform hazard spectrum
VBI	Vehicle-bridge interaction
VD-STMD	Variable damping semi-active tuned mass damper
WDF	Weighted desirability function
WL	Wind load
WRF	Weighted random forest

MATHEMATICAL NOTATIONS

$\mathbf{0}$	Null vector
$\vec{\mathbf{0}}$	Null matrix
$\overline{a}$	Radius of lead-core
$\hat{\boldsymbol{A}}$	System matrix
A_{L}	Total lead-core area of all isolators
$A_{\mathrm{L,iso}}$	Area of the lead-core of a single isolator
$A_{\mathrm{R,iso}}$	Area of the rubber in an isolator
A_{t}	Tributary area of building
$\hat{\boldsymbol{B}}$	Input matrix
B_{s}	Width of building
$\overline{\boldsymbol{C}}$	Damping matrix of structure with control device
$\hat{\boldsymbol{C}}$	Output matrix
$\overline{c}_0$	Damping constant of MR damper at larger velocities
$\overline{c}_1$	Damping constant representing nonlinear roll-off at low velocities
c_1	Damping coefficient of the first story
$c_{1\mathrm{x}}$	Damping coefficient of the first story of structure in the X direction
$c_{1\mathrm{y}}$	Damping coefficient of the first story of structure in the Y direction
c_{b}	Damping coefficient of isolation system
c_{d}	Damping coefficient of TMD
C_{d}	Drag coefficient under wind
c_{d1}, c_{d2}	Damping coefficients of the first and second semi-active damper
$c_{\mathrm{d,max}}$, $c_{\mathrm{d,min}}$	Maximum and minimum values of damping coefficient of TMD
c_{L}	Specific heat of lead
COV	Coefficient of variation
C_{p}	Velocity of wave propagation through soil medium
c_{s}	Damping coefficient of structure
$\boldsymbol{C}_{\mathrm{s}}$	Damping matrix of structure/superstructure

$\boldsymbol{C}_{\mathrm{sx}}$, $\boldsymbol{C}_{\mathrm{sy}}$	Damping matrix of primary structure in the X and Y directions, respectively
d	Number of the values of a random variable
D	Radius of influence beyond which the influence of interpolation points vanishes
$\hat{\boldsymbol{D}}$	Feed forward matrix
$d_{R_{\mathrm{i}(\chi)}}$	Individual desirability function of the estimated i^{th} response quantity
dx_{b}	Change in isolator displacement
dx_{d}	Change in TMD displacement
dx_{N}	Change in top floor displacement
$\boldsymbol{dX}_{\mathrm{s}}$	Vector of change in floor displacements
$d_{(\chi,\chi_{\mathrm{a}})}$	Distance between the point where approximate response is required to the a^{th} design point
$D_{(\chi)}$	Composite desirability function
$D^{*}_{(\chi)}$	Modified composite desirability function
E	Young's modulus of the soil medium
E_{c}	Modulus of elasticity of steel
$E_{\mathrm{h,b}}$	Energy dissipated through the hysteretic behavior of base isolation system
$E_{\mathrm{h,TMD}}$	Energy dissipated through the hysteretic behavior of TMD
E_{i}	Total input energy
E_{k}	Kinetic energy
$E_{\mathrm{k,BI}}$	Kinetic energy in base-isolated structure (excluding the TMD)
$E_{\mathrm{k,BI,p}}$	Predicted kinetic energy in base-isolated structure
$E_{\mathrm{k,TMD}}$	Kinetic energy in TMD
E_{m}	Mechanical energy
$E_{\mathrm{m,p1}}$ - $E_{\mathrm{m,p4}}$	Predicted mechanical energies
E_{s}	Elastic strain energy

$E_{\mathrm{s,BI}}$	Elastic strain energy in base-isolated building (excluding the TMD)
$E_{\mathrm{s,BI,p}}$	Predicted elastic strain energy in base-isolated building
E_{sd}	Cumulative dissipated energy by structural damping
$E_{\mathrm{s,TMD}}$	Elastic strain energy in TMD
E_{TMD}	Energy dissipated by TMD
F_0	Normalized yield strength of isolator
F_0^{X}, F_0^{Y}	Normalized yield strength of isolator in the X and Y directions, respectively
$\bar{f}$	Tuning frequency ratio
$\bar{\boldsymbol{f}}$	Vector of the restoring forces in base isolators
F_{A}	Amplitude of excitation
f_{b}	Force in the base isolators
f_{bx}, f_{by}	Force in the base isolators in X and Y directions, respectively
$\bar{\boldsymbol{F}}_{\mathrm{d}}$	Column vector of forces in dampers
f_{d1}, f_{d2}	Forces in the first and second dampers
$\bar{\boldsymbol{F}}_{\mathrm{exc}}$	Excitation vector for structure with control device
f_{L}	Excitation frequency ratio
f_{MR}	Force in MR damper
F_{s}	Excitation
$\boldsymbol{F}_{\mathrm{s}}$	Excitation vector for the primary structure
f_{s1}	Force in sliding system
f_{w}	Wind force
$\boldsymbol{F}_{\mathrm{w}}$	Column vector of wind forces applied at floor levels
f_{wb}	Wind forces acting at the base slab
f_{wj}	Wind force acting at the j^{th} floor
F_{y}	Yield strength of isolation system
F_{ys}	Yield force of the sliding system

$F_{y,\bar{T}_L}$	Effective yield strength of the lead-core
G_R	Shear modulus of rubber
$H(\bullet)$	Heaviside step function
h_c	Free height of steel column
h_L	Height of the lead-core
$H_{\bar{p}i}(\hat{k}\Delta f)$	Lower triangular matrix that is obtained by Cholesky decomposition of the power spectral density function
H_s	Story height
h_x, h_y	Nondimensional hysteretic displacement components in the X and Y directions, respectively
$\boldsymbol{I}$	Identity matrix
I_c	Moment of inertia of cross-section
I_{KE}	Relative importance factor for kinetic energy
I_{ratio}	Importance factor ratio
I_{SE}	Relative importance factor for elastic strain energy
I_{HZ}	The intensity measure of the hazard under which the reliability is being evaluated
J_1 - J_{12}	Performance criteria
J_{av}	Average performance criteria
k	Total number of input variables in a response surface model
$\bar{\boldsymbol{K}}$	Stiffness matrix of structure with control device
k_0	Stiffness of MR damper at larger velocities
k_1	Stiffness of the first story
k_{1x}, k_{1y}	Stiffness of the first story in the X and Y directions, respectively
k_a	Stiffness of the accumulator
k_b	Stiffness of isolation system
k_{bs}	Post-yield stiffness of sliding system
k_d	Stiffness of TMD
k_i	Initial stiffness of isolation system

$k_{\mathrm{I,iso}}$	Initial lateral stiffness of each isolator
k_{iso}	Lateral stiffness of an isolator
k_{j}	Stiffness of the j^{th} story of structure
k_{s}	Stiffness of primary structure
$\boldsymbol{K}_{\mathrm{s}}$	Stiffness matrix of the structure/superstructure
$\hat{k}_{\mathrm{s}}$	Thermal conductivity of steel
$\boldsymbol{K}_{\mathrm{sx}}$, $\boldsymbol{K}_{\mathrm{sy}}$	Stiffness matrix of parent structure in the X and Y directions, respectively
LS_{i}	The limit state imposed on the i^{th} response quantity
LS_{ji}	The limit state imposed on the i^{th} response quantity under the j^{th} hazard
$LV_{R_{\mathrm{i}}}$	Lower value of reliability below which the case is considered to be unacceptable
$\bar{\boldsymbol{M}}$	Mass matrix of structure with control device
$\mathrm{Max}(\Delta_{\mathrm{j}}^{\sigma})$	Resultant maximum inter-story drift ratio
$\mathrm{Max}(\Delta_{\mathrm{j}}^{\mathrm{x}})$	Maximum inter-story drift ratio in the X direction
$\mathrm{Max}(\Delta_{\mathrm{j}}^{\mathrm{y}})$	Maximum inter-story drift ratio in the X direction
m_{b}	Mass of the base slab
m_{d}	Mass of the TMD
m_{j}	Mass of the j^{th} floor
m_{s}	Mass of the primary structure
$\boldsymbol{M}_{\mathrm{s}}$	Mass matrix of primary structure/superstructure
M_{t}	Total mass of the base-isolated structure
N	Number of stories / number of degrees of freedom
n , A, β , τ	Dimensionless parameters that characterize the Bouc-Wen model
$\bar{n}$, $\bar{\gamma}$, $\bar{\beta}$, $\bar{A}$	Dimensional parameters that characterize the modified Bouc-Wen model used for MR damper
$\hat{n}$	Number of response quantities to be optimized
$\bar{N}$	Number of data points in the discrete time series

N_{alt}	Number of design alternatives
N_{exp}	Total number of experimental design points
N_{iso}	Number of isolators
$N_{\mathrm{R,layers}}$	Number of rubber layers in an isolator
p	Total number of unknown coefficients in the response surface model
$P_{\mathrm{f,Acc}}$	Probability of exceedance of peak acceleration limit state
$P_{\mathrm{f,Drift}}$	Probability of exceedance of peak inter-story drift limit state
$P_{\mathrm{f,IsoDis}}$	Probability of exceedance of peak isolator displacement limit state
$P_{\mathrm{f,Shear}}$	Probability of exceedance of peak normalized base shear limit state
P_{iso}	Vertical load acting on each base isolator
q	Yield displacement of isolation system
Q	Equivalent trinitrotoluene (TNT) charge weight
$\overline{q}$	The number of hazards to be considered in the optimization
$\widehat{R}$	Distance from the charge center
$r_{\mathrm{hz,j}}$, $r_{\mathrm{R,i}}$	Relative importance ratio values assigned for hazards and response quantities
$R_{\mathrm{i,max}}$	Upper value of reliability above which the case is considered to be best
$R_{\mathrm{i}(\chi)}$	Reliability of the structure considering the i^{th} response quantity
R_{rup}	Closest distance to rupture plane
S_{F}	Scaling factor
$S_{\mathrm{rr}}(\hat{z},\omega)$	PSDF of longitudinal fluctuating wind speed
t	Total elapsed time since the beginning of the excitation
T_{b}	Isolation time period
t_{d}	Arrival time of shock wave due to blast
$\overline{T}_{\mathrm{L}}$	Temperature of the lead-core
$t_{\mathrm{R,tot}}$	Total rubber thickness in an elastomeric isolator
$t_{\mathrm{R,layer}}$	Thickness of individual rubber layer in an isolator
t_{s}	Total thickness shim plates in an isolator

t_{shim}	Thickness of an individual shim plate in an isolator
T_{s}	Fundamental time period of structure
u	Input
$\boldsymbol{u}$	Input vector
u_{*}	Shear friction velocity
u_{f}	Fluctuating wind speed
$\bar{\dot{u}}_{\mathrm{g}}$	Peak particle velocity
$\ddot{\boldsymbol{U}}_{\mathrm{g}}$	Vector of ground accelerations
$\ddot{U}_{\mathrm{g}}(t)$	Ground acceleration induced due to underground blast
U_{mean}	Mean wind speed
$U_{\mathrm{mean}}(\hat{z})$	Mean wind speed at height $\hat{z}$
u_{p}	Peak control force
V	Volume of the charge chamber
$\boldsymbol{V}$	Diagonal matrix of the relative weights
v_{b}	Basic wind speed
v_{d}	Voltage supplied to the current driver of the MR damper
$v_{\mathrm{d,max}}$	Maximum voltage level supplied to MR damper
$v_{\mathrm{d,min}}$	Minimum voltage level supplied to MR damper
$v_{\mathrm{f}}(t)$	Fluctuating wind speed (longitudinal) at time t
$V_{\mathrm{P}}^{\mathrm{LS}}$	Limit state of peak normalized base shear
$V_{\mathrm{x}}^{\mathrm{n}}$, $V_{\mathrm{y}}^{\mathrm{n}}$	Normalized base shear in the X and Y directions, respectively
V_{σ}^{n}	Resultant normalized base shear
W_{BI}	The total weight of the base-isolated building
$\hat{w}_{\mathrm{i}}$	Weight assigned for the i^{th} response quantity
W_{t}	Total weight of the base-isolated building
$w_{(\chi,\chi_{\mathrm{a}})}$	Weight function in the moving least squares-based response surface methodology

$\boldsymbol{W}_{(\chi)}$	Diagonal matrix of weight functions at the point where response is to be estimated
$\bar{x}_0$	Initial displacement associated with the accumulator spring stiffness under nominal force
$\bar{\boldsymbol{X}}$	Column vector of displacements for the structure with control device in the X direction
$\dot{\bar{\boldsymbol{X}}}$	Column vector of velocities for the structure with control device in the X direction
$\ddot{\bar{\boldsymbol{X}}}$	Column vector of accelerations for the structure with control device in the X direction
x_{b}	Displacement of the base slab relative to the ground in the X direction
$\dot{x}_{\mathrm{b}}$	Velocity of the base slab relative to the ground in the X direction
$\ddot{x}_{\mathrm{b}}$	Acceleration of the base slab relative to the ground in the X direction
$x_{\mathrm{b,p}}$	Predicted value of the displacement of the base slab relative to the ground
$\dot{x}_{\mathrm{b,p}}$	Predicted value of the velocity of the base slab relative to the ground
$x_{\mathrm{b\text{-}P}}^{\mathrm{LS}}$	Limit state of peak isolator displacement
x_{d}	Displacement of the TMD
$\dot{x}_{\mathrm{d}}$	Velocity of the TMD
$\ddot{x}_{\mathrm{d}}$	Acceleration of the TMD
$\ddot{x}_{\mathrm{g}}$	Ground acceleration in the X direction
$\ddot{x}_{\mathrm{g0}}$	Amplitude of the ground acceleration
$\ddot{x}_{\mathrm{g(i)}}$	Input ground acceleration at time $t_{(\mathrm{i})}$
$\ddot{x}_{\mathrm{g(i+1),estimated}}$	Estimated value of ground acceleration at time $t_{(\mathrm{i+1})}$
x_{j}	Displacement of the j^{th} floor
$\dot{x}_{\mathrm{j}}$	Velocity of the j^{th} floor
$\ddot{x}_{\mathrm{j}}$	Acceleration of the j^{th} floor
$x_{\mathrm{N,PTMD}}$	Peak top floor displacement of the structure with PTMD

$\ddot{x}_{\mathrm{N,PTMD}}$	Peak top floor acceleration of the structure with PTMD
$x_{\mathrm{N,SATMD}}$	Peak top floor displacement of the structure with SATMD
$\ddot{x}_{\mathrm{N,SATMD}}$	Peak top floor acceleration of the structure with SATMD
x_{N} , y_{N}	Top floor displacements in the *X* and *Y* directions, respectively
x_{pm}	Peak actuator stroke
$\ddot{x}^{\mathrm{LS}}_{\mathrm{P\text{-}Max}}$	Limit states of the maximum of peak floor accelerations
x_{red}	Percentage reduction in peak value of displacement response
$\ddot{x}_{\mathrm{red}}$	Percentage reduction in peak value of acceleration response
$\ddot{x}^{\mathrm{LS}}_{\mathrm{RMS\text{-}Max}}$	Maximum of root mean square floor accelerations
$\boldsymbol{X}_{\mathrm{s}}$	Column vector of floor displacements in the *X* direction
$\dot{\boldsymbol{X}}_{\mathrm{s}}$	Column vector of floor velocities in the *X* direction
$\ddot{\boldsymbol{X}}_{\mathrm{s}}$	Column vector of floor accelerations in the *X* direction
$\boldsymbol{X}_{\mathrm{s,min}}$, $\boldsymbol{X}_{\mathrm{s,max}}$	Vectors of predicted displacements of primary structure
$\dot{\boldsymbol{X}}_{\mathrm{s,min}}$, $\dot{\boldsymbol{X}}_{\mathrm{s,max}}$	Vectors of predicted accelerations of primary structure
$x_{\mathrm{s,p}}$	Predicted floor displacement relative to base slab
$\dot{\hat{x}}_{\mathrm{s,p}}$	Predicted floor velocity relative to the ground
$\boldsymbol{X}_{\mathrm{s,p}}$	Column vector of predicted floor displacements relative to base slab
$\dot{\hat{\boldsymbol{X}}}_{\mathrm{s,p}}$	Column vector of predicted floor velocities relative to base slab
$x_{\mathrm{s,PTMD}}$	Peak displacement response of the structure equipped with the PTMD
$\ddot{x}_{\mathrm{s,PTMD}}$	Peak acceleration response of the structure equipped with the PTMD
$x_{\mathrm{s,SATMD}}$	Peak displacement response of the structure equipped with the SATMD
$\ddot{x}_{\mathrm{s,SATMD}}$	Peak acceleration response of the structure equipped with the SATMD
$\boldsymbol{y}$	Output vector
y_{b}	Displacement of the base slab relative to the ground in the *Y* direction
$\dot{y}_{\mathrm{b}}$	Velocity of the base slab relative to the ground in the *Y* direction
$\ddot{y}_{\mathrm{b}}$	Acceleration of the base slab relative to the ground in the *Y* direction

$\ddot{y}_g$	Ground accelerations in the *Y* direction
$Y_{\bar{p}}(\hat{k}\Delta f)$	Complex random numbers
$\boldsymbol{Y}_s$	Column vector of floor displacements in the *Y* direction
$\dot{\boldsymbol{Y}}_s$	Column vector of floor velocities in the *Y* direction
$\ddot{\boldsymbol{Y}}_s$	Column vector of floor accelerations in the *Y* direction
Z	Model matrix of the coded input variables
$\hat{z}$	Height at which the wind speed is to be evaluated
$\bar{z}$	State vector
$\dot{\bar{z}}$	First derivative of state vector with respect to time
Z_d	The model matrix of the coded input variables at the design points
$\bar{z}_i$	State vector at the i^{th} time instant
$\bar{z}_{i+1}$	State vector at the $(i + 1)^{th}$ time instant
$\bar{z}_t$	The state vector, $\bar{z}$, at time t
$Z_{(\chi)}$	The model vector of size $1 \times p$ for a point of interest
α	Post-yield to pre-yield stiffness ratio of isolator
$\bar{\alpha}$, b	Terrain variables
α_s	Thermal diffusivity of steel
β	A column vector of the unknown coefficients
β_0, β_j, β_{jj}, β_{ij}	Unknown coefficients to be evaluated in response surface model
$\beta_{(\chi)}$	A vector of the unknown coefficients for a specific point at which response shall be obtained
ΔC	Uncertainty in stiffness
ΔK	Uncertainty in stiffness
$\Delta^{LS}_{P\text{-}Max}$	Limit state of the maximum of peak inter-story drift ratio
Δt	Small time interval
Δ^{x}_{total}, Δ^{y}_{total}	Total superstructure drift ratio
Δ^{σ}_{total}	Resultant total superstructure drift ratio
$\Delta\psi$	Percentage difference in response quantity when obtained without and with bidirectional interaction

ε	Error
ε_{ik}	Complex Gaussian random number with zero mean
λ	Output or response quantity
$\boldsymbol{\lambda}$	Column vector of the outputs obtained at each design point
λ_d	Vector of the values of a response quantity at the design points
μ_b	Friction coefficient of isolation system
μ_d	TMD mass ratio
μ_{PGA}	Mean value of the peak ground acceleration
μ_Q	Mean value of equivalent TNT charge weight
μ_v	Mean value of basic wind speed
ξ_b	Isolation damping ratio
ξ_d	TMD damping ratio
ξ_s	Damping ratio of primary structure/superstructure
ρ	Air density
ρ_L	Density of lead
ρ_s	Average density of the soil medium
σ_b	Resultant isolator displacement
σ_N	Resultant top floor displacement
$\ddot{\sigma}_N$	Resultant top floor acceleration
σ_{PGA}	Standard deviation of earthquake PGA
σ_Q	Standard deviation of equivalent TNT charge weight
σ_u	RMS control force
σ_v	Standard deviation of basic wind speed
σ_{xm}	RMS actuator stroke
$\sigma_{y,0}$	Effective yield stress of lead at reference temperature
$\sigma_{y,\bar{T}_L}$	Yield stress of lead after a rise in lead-core temperature by $\bar{T}_L$
τ	Dimensionless time
$\phi_{L,iso}$	Diameter of lead-core of an isolator

χ	Vector form of a set of input variables (a column vector of the coded variables)
χ_i , χ_j	Coded (independent) input variables
ψ_{Int}	Peak value of resultant response quantity when bidirectional interaction is considered
ψ_{NoInt}	Peak value of resultant response quantity when bidirectional interaction is not considered
ψ_r	Resultant value of response quantity
ω	Angular frequency of wind
ω_b	Angular frequency of the isolation system
ω_d	Angular frequency of the TMD
ω_L	Angular frequency of excitation
ω_s	Angular frequency of structure
ω_{s1}	Fundamental angular frequency of structure