STRUCTURE PRESERVING AND PARAMETRIC REDUCED ORDER MODELING STRATEGIES FOR SYSTEM SIMULATION

by

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Certificate

This is to certify that the thesis entitled **"Structure Preserving and Parametric Reduced Order Modeling Strategies for System Simulation"**, submitted by **Mohd Abid Bazaz** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor in Philosophy in Electrical Engineering, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

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Abid.

Abstract

The importance of numerical simulations has steadily increased across virtually all scientific and engineering disciplines. In many application areas, experiments have been largely replaced by numerical simulations in order to save costs in design and development. High accuracy simulation requires high fidelity mathematical models which, in turn, induce dynamical systems of very large dimensions. The ensuing demands on computational resources can be overwhelming and efficient model utilization becomes a necessity. It often is both possible and prudent to produce a lower dimensional model that approximates the response of original one to high accuracy. There are many model reduction strategies in use that are remarkably effective in the creation of compact, efficient and high fidelity dynamical system models. Krylov-subspace based reduction is one of the most popular methods for reduced order modeling of very large systems with sizes ranging from tens of thousands to millions. However, one of the main drawbacks of this method is the lack of a proper stopping criterion. In Chapter 3 of this dissertation, we propose a simple and efficient stopping criterion for automated and efficient order selection for Krylov-based reduction techniques. Numerical experiments conducted on some benchmark problems validate the proposed criterion.

Typically, a reduced order model represents a specific instance of the physical system under study, and as a consequence will have high fidelity only for small variations around that base system instance. Significant modifications to the physical model such as geometric variations, changes in material properties, or alterations in boundary conditions generally necessitate generation of new reduced models. Since the generation of a high fidelity reduced model may be comparable in expense to a brief simulation of an instance of the original full order model, the benefits of model reduction will be fully realized only if the parameter dependency found in the original dynamical system can be preserved in some fashion within the

reduced model. This precisely is the goal of Parametrized Model Order Reduction which is one of the thrust areas in research related to model order reduction at present. In Chapter 4 of this dissertation, we present a two-step reduction strategy for differential-algebraic system obtained after finite element discretization of the parametric transient electromagnetic computational problem. For this problem, we assume a parametric dependence which allows a symbolic parametric presence in the Finite Element model. The strategy proposed for this system results in a drastically reduced model with the parametric dependence symbolically preserved, thereby allowing repetitive simulations in the reduced space without the need to refer back to the original system.

The problem of parametric reduction of large systems becomes complicated if the parametric dependence cannot be expressed analytically. This is the case with parameters like geometrical variations which have a direct influence on discretizations and as such do not have have an explicit presence in the Finite Element Models. For parametric reduction of such systems, a framework based on matrix interpolations is usually used. However, one of the main issues in this framework is how to sample the parametric space so as to capture the hidden parametric dependence optimally. In Chapter 5 of this thesis, a numerical measure based on subspace angles is used for quantifying the sensitivity of the model to parametric variations. The relative sensitivity information so obtained is then used for adaptive sampling of the parametric space.

Large second order systems with special structures arise in the study of many physical systems. While reducing such systems, it is desirable that the structure is preserved in the reduced model. Methods based on second-order Krylov subspaces exist for such structure preserving reductions. In Chapter 6 of this thesis, we extend the notion of structure preserving reductions to parametric second order systems with analytically inexpressible parametric dependence. A strategy is developed within the broad framework of matrix interpolations for achieving such reductions.

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