

A Thesis

on

GROUP THEORETIC APPROACH TO THE SOLUTION
OF FLUID MECHANICS EQUATIONS AND RELATED
PHYSICAL SYSTEMS

by

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CERTIFICATE

This is to certify that the thesis entitled
'Group Theoretic Approach To The Solution Of Fluid Mechanics
Equations and Related Physical Systems", being submitted by
Miss Poornima Dewanwala to the Indian Institute of Technology,
Delhi, for the award of the Degree of Doctor of Philosophy
in Mathematics, is a record of bonafide research work carried
out by her. Miss Poornima Dewanwala has worked under my
guidance and supervision and has fulfilled the residential
and other requirements for the submission of this thesis,
which to my knowledge, has reached the requisite standard.
The results contained in this thesis have not been submitted,
in parts or in full, to any other University or Institute
for the award of any degree or diploma.


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