A THESIS ON

A STUDY OF DISCONTINUITIES AND REGULAR FLOWS IN NON-RELATIVISTIC AND RELATIVISTIC MAGNETOGASDYNAMICS

By

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Submitted to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics

1972

ACKNOWLEDGEMENTS

I am extremely grateful to Dr. O.P. Bhutani, Department of Mathematics, Indian Institute of Technology, Delhi, for his valuable guidance and encouragement throughout the preparation of this thesis.

I take this opportunity to express my grateful thanks to Dr. M.K. Jain, D. Phil., D.Sc., Head of the Department of Mathematics, Indian Institute of Technology, Delhi, for his constant encouragement and advice.

I am deeply thankful to Dr. S.D. Chopra, Head of the Department of Mathematics, Kurukshetra University, Kurukshetra, for his constant encouragement and interest in my work.

I also wish to express my sincere thanks to the authorities of the Indian Institute of Technology, Delhi, for providing the research facilities and those of C.S.I.R., India, for the financial grant for doing the present work.

Finally, I thank Mr. D.R. Joshi for his patient typing of this thesis.

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C E R T I F I C A T E

This is to certify that the thesis entitled 'A Study Of Discontinuities And Regular Flows In Non-Relativistic And Relativistic Magnetogasdynamics' being submitted by Miss Shanti Juneja to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by her. Miss Shanti Juneja has worked under my guidance and supervision and has fulfilled the residential and other requirements for the submission of this thesis, which to my knowledge, has reached the requisite standard. The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

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S Y N O P S I S

The thesis is concerned with macroscopic non-relativistic as well as relativistic study of certain problems in the dynamicsof inviscid, non-heat conducting, compressible fluids of infinite electrical conductivity. In all it comprises six chapters. Chapter-I presents an introduction to the entitled thesis. ChaptersII and III are devoted to the determination of flows behind shock waves in non-relativistic magnetogasdynamics. Chapter-IV deals with the classification of solutions of magnetogasdynamic equations and chapters V and VI present relativistic counterparts of the problems dealt with in chapters II and IV respectively. A brief resume of each of the chapters is given below.

Chapter I - Introduction

Here, after giving a brief chronological survey of the available literature relevant to our work in chapters II and III, we have given the fundamental equations of non-relativistic magneto-fluiddynamics. Further, we have given a short historical survey of the available works in Lie group theory and its applications relevant to our work in chapters IV and VI. For ready reference, we have also introduced the notions of Lie groups, Lie derivative and certain other notions and theorems needed in the discussions of chapters IV and VI. Finally, reference is made to chapter V for the fundamental equations and for a brief chronological survey of the related works in relativistic magnetogasdynamics.

Chapter II - Compatibility Conditions for Singular Surfaces In Three-Dimensional Unsteady Magnetogasdynamics

In this chapter, the existence of an arbitrary singular surface $\Sigma(t)$ in an otherwise continuous flow field of three-dimensional unsteady magnetogasdynamics is postulated such that across $\Sigma(t)$ the flow and field variables and hence their space-time derivatives are discontinuous and have finite jumps. Following a procedure similar to that of Coburn (1965) we have, then, found the jumps in the first order space-time derivatives of the flow and field variables along the normal to the shock surface $\Sigma(t)$ assuming that the speed of the shock and the flow ahead of it are known and that the flow and field variables just behind the shock are expressible in terms of these known quantities. The particular cases of contact manifolds and perpendicular shock waves are discussed. For comparison purposes, we have also obtained the relations of interdependence of these jumps via Thomas' technique and these are shown to be identical to those obtained by using the extended form of Coburn's technique. Finally, the advantages of explicit and simultaneous determination of the jumps through Coburn's technique over the other are pointed out.

Chapter III - Determination of Flows Behind Stationary Magnetogasdynamic Shock Waves

Here, we have devised a general procedure for the determination of the gradients of flow and field parameters behind shock waves in three-dimensional steady flows of ideal, non-heat conducting and infinitely electrically conducting gases. Assuming the flow ahead of the shock to be uniform and known, the explicit expressions for the gradients, the vorticity and the curvature of stream-lines just behind the shock surface are determined. Also, mention is made of the advantages of the present technique, which is closest in spirit to that of Morioka (1956), over the available work of Ram (1967,68) on the entitled problem.

Chapter IV - Classification of Solutions of Magnetogasdynamic Equations Using Certain Space Lie Transformation Groups

In this chapter, we have developed a geometric method for determining and investigating certain classes of solutions of the system of non-linear first order partial differential equations governing the three-dimensional unsteady flow of an ideal, non-heat conducting and infinitely electrically conducting gas. More specifically, we have derived the conditions for a solution to be regular with respect to the one-parameter group G of homothetic motions of a three-dimensional Euclidean metric space R_3. Further, it has been shown that if the regularity conditions mentioned above are satisfied then, the problem of obtaining regular solutions reduces to that of solving a system of equations not involving λ in the canonical coordinates (λ, μ, ν) of the subgroup. Some special classes of flows have been investigated and for the particular case of steady-state plane flows it has been shown that the problem of obtaining solutions regular in relation to one-parameter group of homothetic motions of the plane reduces to that of solving a system of ordinary differential equations in the canonical coordinates of the subgroup. In conclusion, a few implications of the steady-state regular flows have been discussed.

Chapter V - <u>Compatibility Conditions for Singular-Hypersurfaces in Relativistic Magnetogasdynamics</u>

After giving, in brief, the chronological survey of the available literature relevant to the present work we have given, in this chapter, the general macroscopic equations of relativistic fluid mechanics. Taking into consideration the locally adiabatic flow of an inviscid, non-heat conducting and infinitely electrically conducting gas in the space-time of general relativity, we have postulated in it the existence of a singular hypersurface Σ_3 of order zero. Following a procedure similar to that used in chapter II, we have, then, determined the explicit jumps in the normal derivatives of the flow and field parameters across Σ_3 assuming that the speed of the shock Σ_3 and the flow ahead of it are known and that the flow variables just behind it are expressible in terms of these known quantities. Finally, the particular case of contact manifolds has been discussed.

Chapter VI - Classification of Solutions of Relativistic Magnetogasdynamic Equations Using Certain Space-Time Lie Transformation Groups

Here, the geometric method developed in chapter IV has been extended to the system of first order non-linear partial differential equations of relativistic magnetogasdynamics. Considering the one-parameter group G of homothetic motions of the space-time of general relativity, solutions of the system of equations referred to above 'regular' in relation to G are determined. It has further been shown that, in case, the conditions of regularity are satisfied, the problem of obtaining regular solutions reduces to that of solving a system of equations not involving λ in the canonical coordinates $(\lambda, \mu, \nu, \chi)$ of the subgroup. Some special classes of flows have been investigated and in conclusion, it has been proved that the world-lines of a flow regular in relation to G are transformed into themselves by finite transformations of the group and that they cannot coincide with the trajectories of the group.

C O N T E N T S