

SOME PROBLEMS ON MHD FLOW OF ELECTRI-  
CALLY CONDUCTING NON-NEWTONIAN FLUIDS.

by

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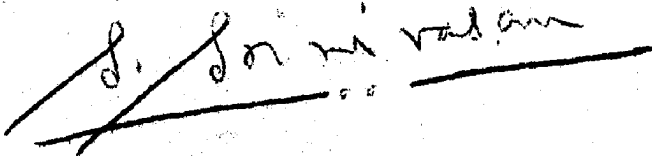
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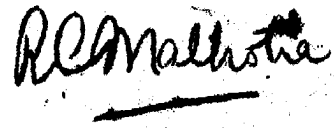
CERTIFICATE

This is to certify that the thesis entitled 'Some problems on MHD flow of electrically conducting non-Newtonian fluids' being submitted by Mr. A. N. Sriramulu to the Indian Institute of Technology, New Delhi for the award of the Degree of Doctor of Philosophy in the Faculty of Engineering is a record of bonafide research carried out by him. Mr. A. N. Sriramulu has worked under our guidance and supervision and has fulfilled the requirements for the submission of this thesis, which to our knowledge has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.



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## Abstract

Fluids having a wide range of visco-elastic properties have been extensively studied by researches over the past two decades. The development of new synthetic materials having application in the process industry have given a strong impetus for such investigations wherein it has become necessary to establish not only the correct production parameters but also to provide explanations for many fundamental problems.

One problem which has especially attracted the attention of research investigators is the study of Tom's phenomenon which accompanies the turbulent flow of solutions of linear polymers in closed conduits. Even in the flow of dilute polymer solutions, which have essentially the characteristics of Newtonian liquids, considerable drag reduction and distinct damping has been reported. This effect has a direct relationship with the stabilising influence that has been observed in laminar flow of a fluid which has some polymer molecules dissolved or suspended in it. It is manifested by an increase of the critical Reynolds number marking the transition from laminar to a turbulent flow. Several explanations for this effect have been hypothesized in literature using both the structural and continuum theory. The explanations in the light of structural theory are based on particulate

effects, such as the number of particles, their size, flexibility etc.; on the other hand the continuum models (macroscopic in nature) state that this drag reduction is due to the elastic properties of the fluid (memory of the flow) manifested by the introduction of polymers or any other substances.

To explain the behaviour of these fluids particularly, Tom's phenomenon, several constitutive equations have been formulated by Reiner (1945), Rivlin (1948), Oldroyd (1950), Rivlin and Ericksen (1955). Using the above constitutive equations the problems in hydrodynamics have been successfully extended to non-Newtonian fluids to study the effect of cross-viscosity and coefficient of elasticity. Walters (1962) studied the stability of fluids with memory. Beard and Walters (1964) examined the two-dimensional flow near a stagnation point. Chan Man Fong and Walters (1965) solved the modified Orr-Sommerfield equation by using the method of Michael. They showed that the coefficient of elasticity destabilises the flow. Chun and Schwarz (1968) examined the plane Poiseuille flow of a second-order fluid and have concluded that the visco-elastic parameter has a destabilizing effect on the flow. Soundalgekar and Pratap Puri (1969) extended the problem further by studying the fluctuating flow past an infinite plate with variable suction for an elastico-viscous fluid.

In the same way as problems in hydrodynamics are extended to magneto-hydrodynamics, the problems in magneto-hydrodynamics have been extended to study the flow of conducting non-Newtonian fluids. Narasimhan (1962) and Kapur (1964) examined the MHD flow of non-Newtonian visco-elastic fluids with a constant pressure gradient in the direction of the main flow. Madan and Agarwal (1969) have considered the unsteady MHD flow of non-Newtonian visco-elastic fluids between two porous flat plates when there exists a time-dependent pressure gradient in the direction of the main flow.

In the present study some aspects of MHD flow of conducting non-Newtonian fluids have been undertaken. Initially the governing equations for the hydro-magnetic flow of non-Newtonian fluids have been derived. Using these the two-dimensional free-convection laminar flow of an incompressible slightly conducting elastico-viscous fluid with time-varying suction is investigated using Kalony's method. It has been shown that an increase in the coefficient of time-varying suction  $A$ , leads to an increase in the skin friction fluctuations. Further for the same value of  $A$ , an increase in the magnetic parameter  $M$  leads to an increase in the amplitude and a decrease in the

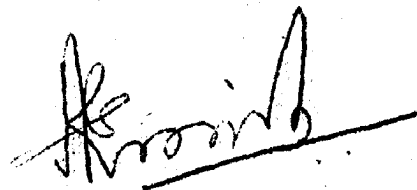
phase lead of skin friction. Also it has been observed that the amplitude of skin friction increases with an increase in the coefficient of elasticity. Expressions for the temperature, the rate of heat transfer, the velocity and the skin friction for both high and low values of the frequency and the magnetic parameter have been obtained.

The second problem that has been examined is the conditions of applicability of Squire's theorem for the stability of parallel flows to that of a conducting Rivlin-Ericksen fluids. It has been found that for slightly visco-elastic fluids which do not belong to this class, Squire's theorem can be satisfied under a set of transformations. It is shown that the effect of visco-elasticity on in-plane and out-of-plane disturbances may be different; one may lead to stabilization and the other to destabilization. Further it is shown that the method of solution that has been used for in-plane analyses can also be applied for three-dimensional analyses.

The last problem studied is the two-dimensional MHD stability of fluids with short relaxation times in the presence of coplanar magnetic field. In this study, Walter's technique has been used. The modified Orr-Sommerfeld equation is solved by using the method developed by Michael (1964) in his discussion of the same problem for a dusty gas. It is found that the coefficient of elasticity destabilizes the flow of electrically conducting non-Newtonian fluids.

The following papers have been published/print  
under the present study:

1. Sriramulu, A.N., and Malhotra, R.C., Two-dimensional MHD stability of elastico-viscous fluid, Tensors, N. S., 22(1971), 98-102.
2. Sriramulu, A.N., On Squire's theorem for electrically conducting visco-elastic fluids, Tensors, N. S., 23(1972), 211-212.
3. Sriramulu, A.N., and Malhotra, R.C., On free-convection laminar flow of slightly conducting elastico-viscous fluid over a porous wall. Indian Journal of Physics, 46 (1972) pp. 101-103.
4. Sriramulu, A.N., On the MHD Stability of plane Poiseuille flow of an electrically conducting elastico-viscous fluid, (Accepted for publication in Letters in Applied Engineering Sciences, November-December issue 1974).



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