

A Thesis on
SOME PROBLEMS OF INFORMATION TRANSMISSION
OVER DISCRETE MEMORYLESS CHANNELS .

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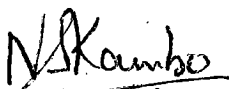
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CERTIFICATE

This is to certify that the thesis entitled "Some problems of information transmission over discrete memoryless channels", which is being submitted by Shri Samar Singh to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy (Mathematics), is a record of bonafide research work. Shri Samar Singh has worked for the last 4 years under my guidance and supervision.

The thesis has attained the standard fulfilling the requirements of the regulations relating to the degree. The results in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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