SOME NEW ALGORITHMS FOR THE STATE ESTIMATION AND CONTROL OF INTERCONNECTED SYSTEMS

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CERTIFICATE

This is to certify that the thesis entitled "Some New Algorithms for the State Estimation and Control of Interconnected Systems" being submitted by Goshaidas Ray, for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work he has done, during the period from March 1978 to September 1981 under my supervision. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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LIST OF PRINCIPAL SYMBOLS

A	state transition matrix
$\bar{A}$	state transition matrix of transformed system
B	control transition matrix
$\bar{B}$	control transition matrix of transformed system
C	measurement matrix (discrete time system)
$\bar{C}$	measurement matrix of transformed system (discrete time system)
d	constant disturbance vector
$e(k)$	innovation sequence of $\{y(k)\}$
$e(t)$	innovation process of $\{y(t)\}$
E	expectation operator
F	state distribution matrix
$\bar{F}$	state distribution matrix of transformed system
G	control distribution matrix
$\bar{G}$	control distribution matrix of transformed system
H	measurement matrix (continuous time system)
$\bar{H}$	measurement matrix of transformed system (continuous time system)
I	identity matrix
$I(u)$	performance index (for stochastic discrete time regulator problem)
$\bar{I}(u)$	performance index of transformed system (for stochastic discrete time regulator problem)

$I_d(u)$	performance index (for deterministic discrete time system)
I_p	identity matrix of dimension p
$J(u)$	performance index (for stochastic continuous time regulator problem)
$\bar{J}(u)$	performance index of transformed system (for stochastic continuous time regulator problem)
$J_a(u)$	performance index of augmented system
k	discrete interval counter
k_f	final time
$K(k)$	Kalman gain matrix
$\bar{K}(k)$	Kalman gain matrix of transformed system
$K(t)$	filter gain matrix
$\bar{K}(t)$	filter gain matrix of transformed system
$L(k), L(t)$	feedback gain matrices
$\bar{L}(k), \bar{L}(t)$	feedback gain matrices of transformed system
$L_e(k), L_{a2}(t)$	output feedback gain matrices
$L_x(k), L_{a1}(t)$	state feedback gain matrices
m	dimension of control and disturbance vector
$M(k)$	solution of the matrix Riccati difference equation
$M(k_f), M(t_f)$	terminal condition
$M(t)$	solution of the matrix Riccati differential equation
n	system order
N	measurement noise covariance matrix
0_p	null matrix of dimension p
p	dimension of measurement vector

$P(k/k-1)$	prediction error covariance matrix
$P(t)$	error covariance matrix
$\overline{P}(k/k-1)$	prediction error covariance matrix of the transformed system
Q	positive semidefinite state weighting matrix
Q_d	positive semidefinite incremental output weighting matrix
R	positive definite control weighting matrix
R_d	positive definite incremental control weighting matrix
S	non-singular transformation matrix (for continuous time system)
t_f	final time
T	non-singular transformation matrix (for discrete time system)
$T_1, T_2, \overline{T}_1, \overline{T}_2$	orthogonal transformation matrices
$u(k)$, $u(t)$	control vectors
$u_d(k)$	incremental input vector
$U(k/k), \overline{U}(k/k)$	square root matrices of the error covariance matrices $P(k/k)$ and $\overline{P}(k/k)$ respectively
$v(t)$	measurement noise vector
V	measurement noise covariance matrix
$w(k), w(t)$	input noise vectors
W	input noise covariance matrix (for continuous time system)

$x(k)$, $x(t)$	state vectors
$x(0)$	initial state vector
$\hat{x}(k/k-1)$	predicted estimate of the state vector $x(k)$
$\hat{x}(t)$	estimate of the state vector $x(t)$
$\bar{x}(k)$, $\bar{x}(t)$	state vectors of transformed system
$x_d(k)$	incremental state vector
$X(0)$	covariance matrix of the initial state
$y(k)$, $y(t)$	measurement vectors
$\hat{y}(k/k-1)$	predicted estimate of $y(k)$
$\hat{y}(t)$	estimate of the measurement vector $y(t)$
Δ	input disturbance matrix (for discrete time system)
$\bar{\Delta}$	input disturbance matrix of transformed system
Δt	sampling interval
$\eta(k)$	output disturbance vector
Γ	input disturbance matrix (for continuous time system)
$\bar{\Gamma}$	input disturbance matrix of transformed system
Ω	input noise covariance matrix (for discrete time system)

LIST OF COMPUTATIONAL SYMBOLS

Number of major computer operations involved in the Kalman-Bucy filter :

N_{nc}^{KE}	based on the general state variable model
N_{c}^{KE}	based on the WA-canonical state variable model

Number of major computer operations involved in the Chandrasekhar type filter algorithm (continuous time system):

N_{nc}^{CE}	based on the general model
N_{c}^{CE}	based on the canonical model

Number of major computer operations involved in the decentralized filter (continuous time system) :

N^{KDE}	based on the Kalman-Bucy filter algorithm
N^{CDE}	based on the Chandrasekhar type filter algorithm

Total number of major computer operations involved in implementing the control law based on the general model (continuous time system) :

N_{nc}^{KC}	using the conventional control algorithm
N_{nc}^{CC}	using the Chandrasekhar type control algorithm

$\bar{N}_{nc}^{KC}$ using the conventional (P + I) control algorithm

$\bar{N}_{nc}^{CC}$ using the Chandrasekhar type (P + I) control algorithm.

Number of major computer operations involved in implementing the control law based on the canonical model (continuous time system) :

N_{c}^{KC} using the conventional control algorithm

N_{c}^{CC} using the Chandrasekhar type control algorithm

$\bar{N}_{c}^{KC}$ using the conventional (P + I) control algorithm

$\bar{N}_{c}^{CC}$ using the Chandrasekhar type (P + I) control algorithm

Number of major computer operations involved in the Kalman filter (discrete time system) :

M_{nc}^{KE} based on the general state variable model

M_{c}^{KE} based on the WA-canonical state variable model

Number of major computer operations involved in the square root filter (discrete time system) :

M_{nc}^{SE} based on the general model

M_{c}^{SE} based on the WA-canonical model

Number of major computer operations involved in the Chandrasekhar type filter (discrete time system) :

M_{nc}^{CE} based on the general state variable model

M_{c}^{CE} based on the WA-canonical state variable model

Number of major computer operations involved in the decentralized filter (discrete time system) :

M^{KDE}	based on the Kalman filter algorithm
M^{SDE}	based on the square root algorithm
M^{CDE}	based on the Chandrasekhar type filter algorithm
M^{KAD}	based on adaptive Kalman filter algorithm of Sage and Husa

Number of computations involved in implementing the Control law based on the general model (discrete time system) :

M_{nc}^{KC}	using the Kalman filter based control algorithm (centralised)
M_{nc}^{SC}	using the square root filter based control algorithm (centralised)
M_{nc}^{CC}	using the Chandrasekhar filter based control algorithm (centralised)
$\bar{M}_{nc}^{KC}$	using the Kalman filter based (P + I) control algorithm (centralised)
$\bar{M}_{nc}^{SC}$	using the square root filter based (P + I) control algorithm (centralised)
$\bar{M}_{nc}^{CC}$	using the Chandrasekhar filter base (P + I) control algorithm (centralised)

Number of major computer operations involved in implementing the control law based on the WA-canonical model (discrete time system) :

M_c^{KC} using the Kalman filter based control algorithm (centralised)

M_c^{SC} using the square root filter based control algorithm (centralised)

M_c^{CC} using the Chandrasekhar filter based control algorithm (centralised)

$\bar{M}_c^{KC}$ using the Kalman filter based (P + I) control algorithm (centralised)

$\bar{M}_c^{SC}$ using the square-root filter based (P + I) control algorithm (centralised)

$\bar{M}_c^{CC}$ using the Chandrasekhar filter based (P + I) control algorithm.

ABSTRACT

The thesis is concerned with the problems of state estimation and control of linear time invariant interconnected systems. We have studied both continuous and discrete time systems and have examined, in detail, the computational requirements of several algorithms for the solution of these problems.

In the first part of the thesis, we have considered the continuous time state estimation and control problems and have developed some new algorithms by making use of a simplified observable canonical state variable model. We have shown that the computational requirements of the Kalman-Bucy and the Chandrasekhar type filtering algorithms based on this canonical model are significantly less than the requirements of the existing algorithms. We have also indicated the possibility of obtaining a simple suboptimal stable decentralized filter on the basis of the chosen canonical model. We have illustrated the results numerically by considering a two-area power system. We have also simulated this system in order to demonstrate the effectiveness of the proposed canonical Chandrasekhar algorithm. We have next obtained the canonical control algorithms for the LQG problems for systems with zero mean white Gaussian state disturbances and for systems with known constant plus zero mean white Gaussian state disturbances. We have proposed the use of the canonically transformed state estimate for implementation of the proportional

and also of the 'proportional plus integral feedback' control laws. We have again illustrated the results by considering the two-area power system.

In the second part, we have studied the state estimation and the three control problems of the discretized version of the interconnected systems. In addition to the Kalman and the Chandrasekhar type filtering algorithms, we have studied the square root filtering algorithm also. We have again shown a significant computational advantage of the transformation of the state model to the simplified observable canonical form for all the three forms of the filtering algorithm. In addition, the canonical model can be used to obtain reliable decentralized filters. We have then proposed some new optimal proportional state and 'proportional state plus summation output feedback' control algorithms on the basis of the canonical state model. All the results have been illustrated using the two-area power system problem. We have also used this system in order to illustrate some differences between the solutions of the continuous time problems and their discretized versions.

CONTENTS

Page