STABLE METHODS FOR SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

By

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C E R T I F I C A T E

This is to Certify that the Thesis entitled "Stable Methods for Second Order Ordinary Differential Equations" which is being submitted by Mr.Rakesh Goel for the award of Doctor ofPhil:sophy(Mathematics) to the Indian Institute of Technology, Delhi is record of bonafide research work. He has worked for the last four years under our guidance and supervision.

The Thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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A B S T R A C T

The work presented in the thesis entitled 'Stable Methods for Second Order Ordinary Differential Equations', is divided into five chapters.

A general discussion of numerical methods for solving second order initial value problems is given in the first chapter.

In Chapter II, a class of one step methods for solving the differential equation $y'' = f(x,y,y')$ is discussed. A method of $O(h^{2p})$ in this class involves derivatives of y of order i for $i = 1,2,\ldots,p+1$. It is self starting, stable in the sense of Rutishauser, and P-stable for second order periodic initial value problems $y'' = f(x,y)$, $y(x_o) = y_o$, $y'(x_o) = y'_o$. A bound for the global error is obtained for this class of methods. It has a small error constant. An algorithm for controlling the global error by controlling the local error per unit step has been suggested for this class of methods. Numerical comparisons are given in the end of this chapter.

In Chapter III, a fourth order numerical method for solving the general second order differential equation $y'' = f(x,y,y')$, $y(x_o) = y_o$, $y'(x_o) = y'_o$ has been derived. This method is self starting, P-stable (when applied to $y'' = f(x,y)$), and it involves only two function evaluations per step. An important feature of the formula is that

ability' is obtained even though equations including y' can also be treated, since most of the previous formulae with this property have been applied only to $y'' = f(x,y)$. However, when y' is not present calculations get further simplified. Easily calculable and simple truncation error estimates are given, which can be used for automatic error control in computations. The method is illustrated on numerical examples.

In Chapter IV, two Nyström type methods for the integration of second order differential equations $y'' = f(x,y)$, $y(x_o) = y_o$, $y'(x_o) = y'_o$ are obtained. The formulae are derived in two steps. In the first step we apply the Lobatto quadrature formula with k nodes to the integral form of the initial value problem. This gives one equation for the unknown y_{n+1} which involves k-2 unknown y-values at non-step points. In the second step we express these y-values at k-2 non-step points in terms of these values and y_{n+1}. Thus we get a system of k-1 equations involving y_{n+1}. Hence, at each integration step, generally, a non-linear system of s(k-1) equations has to be solved if $y \in R^s$. Methods of order six and eight which correspond to $k = 4$ and $k = 5$ respectively have been obtained. These methods are shown to be P-stable. Numerical examples are also given in the end of the chapter.

In the last chapter, a sixth order P-stable symmetric four step method for periodic initial value problem $y'' = f(x,y)$, is derived. It requires three function evaluations per step. The method can be considered as a stabilized modification of Lambert and Watson's sixth order implicit method which has a finite interval of periodicity. The method is illustrated on numerical examples.

CONTENTS