

**GEOPHYSICAL FLOW SIMULATIONS WITH  
GEODESIC HEXAGONAL MESHES**

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# **GEOPHYSICAL FLOW SIMULATIONS WITH GEODESIC HEXAGONAL MESHES**

by

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Department of Mathematics

Submitted

in fulfillment of the requirements of the degree of  
**Doctor of Philosophy**

to the



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# Certificate

This is to certify that the thesis entitled “**Geophysical flow simulations with geodesic hexagonal meshes**” submitted by “**Mr. Sarvesh Kumar Dubey**” to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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*Sarvesh Kumar Dubey*

# Abstract

Numerical weather prediction (NWP) modelling using an icosahedral-hexagonal (IcoHex) grid offers a very attractive framework because of being quasi-uniform and free from pole problem. The work presented in this thesis is a smaller piece of a bigger task, which involves combining conservative finite volume and mimetic finite difference schemes to solve global circulation of atmosphere in hydrostatic regime using IcoHex grid and Arakawa C-grid staggering.

We develop a new model “DYNAMICO” on icosahedral grids with the help of already existing novel numerical schemes, viz. finite volume scheme for mass budget, energy conserving finite difference discretization of Coriolis term, mimetic vector operators (these operators mimic the properties of the underlying continuous operators in discrete sense), vertical structure of AGCM LMDZ and 4<sup>th</sup> order Runge Kutta (RK4) time integration scheme etc.

Further, we propose two discrete gradient operators for sub-grid reconstructions: the first one is a finite difference gradient and the other gradient is based on Green-Gauss theorem. With a numerical convergence study, we show that these gradients are inconsistent in finite volume sense until and unless a grid modification or optimization is employed. We suggest a simple grid modification and an iterative grid optimization to get rid off this inconsistency. Later on, we use these discrete gradients to propose two second order accurate advection schemes: Semi-Lagrangian

type Finite Volume Scheme (SLFV) and Method of Lines based Finite Volume Scheme (MLFV). On one hand, SLFV is a ‘remap-type’ advection scheme, which approximates tracer mass traversing through edges in the form of parallelograms and employs a local linear approximation to avoid complexities of the conditional branching. On the other hand, MLFV uses total variation diminishing (TVD) RK3 scheme to integrate instantaneous tracer fluxes. We use a slope limiter to achieve positive-definiteness in both the schemes. These slope limited schemes are then compared with their flux-corrected transport (FCT) counterparts to assess the viability of slope limiters. We extend SLFV scheme to three dimensions (3D) via alternate direction splitting and Van Leer I scheme in the vertical. We validate the performance of SLFV and MLFV using a few recently proposed idealized test cases.

To design higher order advection schemes on IcoHex grid, we approximate tracer profile with bi-quadratic sub-grid reconstructions in GSTC 2D local coordinates by weighted least square approach. Later on, these bi-quadratic sub-grid reconstructions are utilized to implement FF-CSALM and its two flux-area simplifications on IcoHex grid. We examine the performance of these advection schemes using linear tracer transport test cases.

We also assess the performance of DYNAMICO for a few idealized test cases, which include dry baroclinic instability and famous climate benchmark test case of Held and Suarez.

Finally, we present preliminary results of a short term real simulation with orography and idealized long term simulations for an aqua planet to test the coupling of DYNAMICO with the physics of LMDZ5. This coupling is still in an initial phase and requires considerable work involving intense numerical experimentation with DYNAMICO.

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