

**STUDIES IN
PARAMETER ESTIMATION OF CONTINUOUS-TIME
SYSTEMS USING HERMITE POLYNOMIALS**

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for the award of the degree of
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Dedicated to

My Son

Nirmal

CERTIFICATE

This is to certify that the thesis entitled "**Studies in Parameter Estimation of Continuous-time Systems using Hermite Polynomials**" being submitted by **Abraham T. Mathew** to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a record of the bonafide research work carried out by him under my supervision and guidance. The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.



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ABSTRACT

During the past two decades, a great deal of attention has been paid to the use of classical orthogonal polynomials for identification of dynamical systems. The elegant approximation method, using the orthogonal polynomials and the associated operational matrix of integration will result in a set of algebraic equations which can be solved simultaneously to get an estimate of the unknown parameters of the system from the input-output data. The study begins with an investigation into the recursive estimation of the Hermite series expansion coefficients for continuous functions. A comparative study between Laguerre polynomials and Hermite polynomials has been made to look into the numerical properties. These algorithms have been used subsequently in developing recursive identification algorithms for different types of systems. The two step scheme for estimating the parameters of the dynamical system from input-output data has been implemented using the recursive least square estimation algorithms. Extensive simulation have been done to ascertain the feasibility of the approach. The recurrence relations of the Hermite polynomials have facilitated the derivation of recursive algorithms for a class of linear time varying systems. Parameter estimation of nonlinear systems has been obtained through the generation of auxiliary data. The study has been extended to the parameter estimation of models expressed in state-space formulation. The proposed multivariable least square algorithm has been developed by exploiting the redundancy in the estimation of the expansion coefficients using the recursive least square algorithm. The effect of outliers in measurements has also been studied. A robust approximation scheme using Hermite polynomials is reported for the first time. A class of robust identification algorithms have been developed using the above robust approximation algorithms. The derivation of this robust scheme will pave the way for further research in this direction. The general observation is that, the Hermite polynomial can also be used for developing recursive algorithm. There is ample scope for further investigation in the use of orthogonal functions in system studies.

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