THEORY AND ANALYSIS OF PIECEWISE LINEAR RESISTIVE NETWORKS

by

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A B S T R A C T

This thesis is concerned with the theory and analysis of piecewise linear resistive networks. Such networks are characterised by equations of the form

$$\underline{f}(\underline{x}) = \underline{y} \qquad \ldots(1)$$

where $\underline{f}$ is a continuous piecewise linear function mapping R^n into itself, $\underline{x}$ is a vector of network variables and $\underline{y}$ is a vector of inputs. The $\underline{x}$-space is divided into a finite number of regions, r, of dimension n , separated by a finite number of (n-1) dimensional hyperplanes, and in any region R_m, equation (1) is of the form

$$\underset{\sim}{J}^{(m)}\underline{x} + \underline{w}^{(m)} = \underline{y} \qquad \ldots(2)$$

where $\underset{\sim}{J}^{(m)}$ is the constant Jacobian matrix in R_m and $\underline{w}^{(m)}$ is a constant vector. The subject matter of this thesis is divided into nine chapters.

The first chapter is introductory in nature. It surveys the developments in piecewise linear resistive networks and establishes a framework of definitions and notations used in the remaining chapters.

The second chapter deals with the so called 'Corner problem' in piecewise linear network analysis. Katzenelson [25] developed a method for determining the solution $\underline{x}^{(f)}$ corresponding to an input $y^{(f)}$ of a piecewise

linear resistive network containing two terminal resistors characterised by strictly monotonic curves. His method consists of choosing an initial guess $\underline{x}^{(o)}$ in an arbitrary region in the $\underline{x}$-space and then tracing the inverse image or solution curve of the line L joining $\underline{y}^{(o)}$ to $\underline{y}^{(f)}$. This method was later extended by Kuh and Hajj [8] to networks having multiple solutions. An important problem that arises in this method is that whenever the solution curve hits a corner point, i.e., a point lying on more than one boundary hyperplane, it is necessary to find out in which of the other regions containing the corner point the solution curve lies. Fujisawa and Kuh [26] have developed a perturbation technique for solving this problem. Their technique is however useful only when the extension of the solution curve lies inexactly one of the other regions meeting at the corner. A method is proposed in the second chapter which enables determination of the extensions of the solution curve in more than one new region. The method involves determination of the signs of the Jacobian determinant and of cofactors taken one per boundary hyperplane per region.

In the third chapter Fujisawa and Kuh's [26] result which states that while tracing the solution curve, $\underline{y}$ changes direction in neighbouring regions if and only if the Jacobians of these regions have opposite signs is shown to be true at a corner point as well, provided the corner point is not a branch point. If it is a branch point, this result fails. In such cases, it is shown that the sign of a certain cofactor in addition to the sign of the determinant of the Jacobian of each of the two regions is required. This result is shown to simplify to Fujisawa and Kuh's result in the case of neighbouring regions. The corner algorithm developed in chapter two together with the present results are then used to obtain general input-output plots of piecewise linear resistive networks.

In the fourth chapter, the properties of piecewise linear resistive networks excited by more than one time varying inputs are studied. (Such networks are called A.C.networks.) The corner algorithm of chapter two is extended to A.C.networks and a systematic method for tracing the solution curves of such networks is discussed and illustrated.

It is often of considerable importance to know if a given resistive network has a unique solution for all possible inputs. In the fifth chapter, necessary and sufficient conditions for unique solvability i.e., for the mapping $\underline{f}$ to be a global homeomorphism, are developed. These conditions are (i) the Jacobian determinant is nonzero and has the same sign in all regions of the $\underline{x}$ space, (ii) the C-condition is satisfied at each corner point. The C-condition (Corner Condition) is said to be satisfied at a corner point if and only if the solution curve corresponding to a line in the $\underline{y}$-space has exactly two extensions at that point. Tests for the C-condition involving the evaluation of Jacobian determinants and their cofactors are also developed in this chapter.

In the sixth chapter the properties of networks with unique solutions for large inputs are studied. Such networks have at least one solution for any given input. Also they have unique solution for all inputs if the Jacobian determinant is nonzero and has the same sign in all regions of the $\underline{x}$-space.

In the seventh chapter, the two global inverse function theorems developed in the fifth and sixth chapters are used to develop two global implicit function theorems useful for piecewise linear functions. The latter are then used to find sufficient conditions for the existence of output controlled input-output plots.

In the eighth chapter computer implementation of the input-output plots, tests for unique solvability, using the corner algorithm developed in chapter two is discussed.

Finally in the ninth chapter the results of the thesis are summarised and the scope for further research is indicated.

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