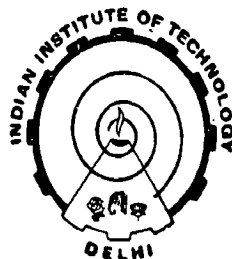


ELASTO-PLASTIC FINITE ELEMENT ANALYSES OF ROCK SLOPES

by

O. P. AILAWADI

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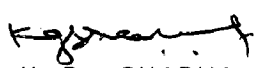


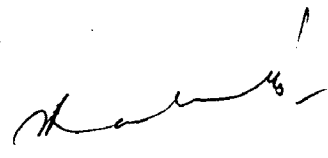
DEPARTMENT OF CIVIL ENGINEERING
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CERTIFICATE

This is to certify that the thesis entitled, "ELASTO-PLASTIC FINITE ELEMENT ANALYSES OF ROCK SLOPES" being submitted by Mr. O.P. Ailawadi to the Indian Institute of Technology, Delhi for the award of the degree of DOCTOR OF PHILOSOPHY is a record of the bonafide research work carried out by him. Mr. Ailawadi has worked under our guidance for the submission of this thesis which to our knowledge has reached the requisite standard.

The thesis or any part thereof has not been submitted to any other university or institute for the award of any degree or diploma.


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No superlative will be able to do full justice in expressing my feelings of compassion towards my wife Promila and daughters Meetu and Geetu for their immense patience and affection which provided me the necessary impetus to work on this thesis.

A handwritten signature in cursive script that reads "O.P. Ailawadi". The signature is written in black ink and is positioned above the printed name.

O.P. Ailawadi

ABSTRACT

Rock cuts have considerable importance because of their high excavation costs, and also as a potential source of damage to installations located in its vicinity. Therefore, a rational design procedure is warranted for their safety. The excavation of a rock slope involves unloading of the original rock mass. None of the methods of slope stability analyses except the finite element method takes into consideration the effect of unloading; moreover, the conventional methods suffer from one drawback or the other. Stability charts are many a times used in the preliminary stages for quick analyses of slopes, in order to study a number of alternative proposals in a project. At present, no stability charts are available for rock slopes prepared on the basis of finite element method. Further, since most of the rocks exhibit non-linear strength envelope, there is a dire need to have stability charts prepared on the basis of some appropriate non-linear failure criterion as well.

Keeping in view the above facts, a systematic study was planned to

- (i) first verify the influence of various parameters (such as E , ν , K_0 and B/H ratio) involved in the slope stability by finite element analyses,
- (ii) conduct elasto-plastic finite element slope stability analyses using the conventional Mohr-Coulomb criterion and compare the results with those available in literature for limit equilibrium method, and finally to prepare stability charts based on finite element method,
- (iii) conduct slope stability analyses, with a suitable non-linear failure criterion (Hoek-Brown criterion in the present study),

- using limit equilibrium method,
- (iv) propose a pair of non-dimensional factors pertaining to the non-linear strength criterion on the basis of which the stability charts could be presented, and
 - (v) conduct elasto-plastic finite element slope stability analyses using the non-linear failure criterion and compare the results with those of limit equilibrium method. Finally, to prepare stability charts for quick analyses.

For the purpose of conducting the above study, two computer programs were developed (i) for the elasto-plastic finite element analyses in which two failure criteria, viz. the conventional Mohr-Coulomb failure criterion and the recently proposed Hoek-Brown failure criterion, were incorporated, and (ii) for the analyses by limit equilibrium method with Bishop's simplified approach using Hoek-Brown failure criterion.

The process of excavation has been simulated in the computer program for the finite element analysis. Elasto-viscoplasticity has been used as an artifice to obtain elasto-plastic solution. A 2 x 2 Gauss - Legendre quadrature has been employed for numerical integration. In the computer program pertaining to the limit equilibrium method, provision has been made for automatic search for a critical toe circle and to list the coordinates of the centres of these circles on the print out.

After verifying that the parameters E , ν , K_0 and B/H do not affect the factor of safety significantly, further parametric study was carried out corresponding to six different slope angles ($\alpha=30^\circ$, 45° , 60° , 70° , 80° and 90°) by varying only the other parameters of influence. In the first phase of study, Mohr-Coulomb failure

criterion was used in the finite element analysis. The determination of factor of safety was done by reducing the strength of the material by a strength reduction factor. For each of the above slopes, five to six strength reduction factors (trial factors of safety) were tried to obtain the value of the factor of safety. The parametric study was carried out in a systematic manner by choosing the properties in such a way that suitable values of $\lambda'_{c\phi}$ (which is inverse of the Janbu's non-dimensional factor $\lambda_{c\phi} = \frac{\gamma H \tan \phi}{c}$ where γ, c and ϕ have the usual meaning and H is the height of slope) were obtained for having a number of points suitably spaced for the preparation of stability charts. For each slope angle, at least five different sets of material properties were used to conform to the required $\lambda'_{c\phi}$ value.

The factors of safety obtained from the results of finite element analyses were compared with those of limit equilibrium method and it has been observed that the limit equilibrium method, in general, underestimates the factor of safety of steeper slopes. For instance, for $\alpha=90^\circ$, the maximum amount of underestimation is 40.7 per cent. For flatter angles, the amount of underestimation is lesser. Sometimes, for $\alpha \leq 45^\circ$, the limit equilibrium method gives unsafe results. Also, the results of factor of safety obtained from the finite element method and limit equilibrium method, are the same at some values of $\lambda'_{c\phi}$.

Stability analyses have also been carried out by limit equilibrium method using Hoek-Brown criterion for slope angles

of 30° , 45° , 60° , 70° , 80° , and 90° . From the results of analyses on about 370 slopes, it has been possible to evolve a pair of non-dimensional factors, $\lambda_{ms} = \frac{\sigma_c s}{\gamma H m}$ and $S'_n = \frac{\sigma_c s^{1/2}}{\gamma H F_s^{3/2}}$ (where σ_c is the uniaxial compressive strength of the material, m and s are the material constants of Hoek-Brown criterion, F_s is the factor of safety and S'_n is the proposed stability number in terms of Hoek-Brown criterion) which are uniquely related through the slope angle. With the help of this unique relation, it is possible to obtain the results of stability number and hence the factor of safety for a particular set of material properties. This is useful for presentation of stability charts on the basis of Hoek-Brown criterion. Further, it has been possible to propose approximate values of coordinates of the centres of critical circle, for use with Hoek-Brown criterion in limit equilibrium method. Such guide lines are not available at present and consequently, large number of iterations are required to be done before a critical toe circle is located. Interestingly, these coordinates of centres for use with Hoek-Brown criterion are functions of λ_{ms} and α (instead of ϕ and α , as in the case of Mohr-Coulomb criterion).

The finite element analysis using Hoek-Brown criterion has also been carried out for $\alpha=30^\circ$ to 90° and the results of limit equilibrium method and those of finite element method are compared. The limit equilibrium method sometimes overestimates and sometimes underestimates the factor of safety depending upon the slope angle and the value of λ_{ms} . For flatter slope, the tendency for

Overestimation is more e.g. for $\alpha=30^\circ$, it is as high as 87.7 per cent. For steeper slope, the tendency for underestimation is more, the maximum amount being 49.9 per cent for $\alpha=90^\circ$. However, for different slope angles, at some specific values of λ_{ms} , the results of the two methods may be same.

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