STATIC AND DYNAMIC STABILITY ANALYSIS OF NONLINEAR ELASTIC CONCRETE BEAM-COLUMNS

by

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CERTIFICATE

This is to certify that the Thesis entitled, **Static and Dynamic Stability Analysis of Nonlinear Elastic Concrete Beam-Columns,** being submitted by **Mamta Rani** to the Indian Institute of Technology Delhi for the award of degree of **Doctor of Philosophy** is a bonafide record of research work carried out by her under our supervision and guidance. The Thesis, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this Thesis have not been submitted, in part or full, to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

Beam-columns are subjected to the simultaneous action of axial compressive forces and flexural moments or transverse forces. Euler method is valid for analysing the static stability of conservative structures only. It predicts infinitely high resistance to static buckling under tangential follower forces. Loss of dynamic stability of such nonconservative structures occurs by flutter at critical magnitudes of follower forces. This dynamic instability occurs at vanishing lateral displacement and infinitely high natural frequency. In contrast, beam-columns under critical conservative forces exhibit vanishing natural frequency and the corresponding dynamic in stability called divergence. Another dynamic instability called parametric resonance occurs under pulsating axial forces as and when the forcing frequency either equals or is twice the natural frequency. Thus, parametric resonance is either fundamental or T2 regular subharmonic resonance. Damped beam-columns are less vulnerable to all these types of dynamic instabilities. Further developments include initial post-buckling theory, active control of dynamic instabilities, flexural torsional buckling, stability criterion of Liapunov, catastrophic theory, complexity theory, etc. Static and dynamic stability is associated with the stability or otherwise of the solutions of the underlying differential equations of equilibrium and of motion respectively. Most of these investigations pertain to physically linear columns or beam-columns undergoing finite lateral displacements. The axial vibrations are generally ignored.

The behaviour of concrete structures till failure is determined by progressive damage and cracking of concrete in tension, yielding of steel reinforcement bars, debonding at concrete–rebar interface, shear transfer across cracks, tension stiffening, nonlinear inelastic behaviour upto failure of concrete under multi axial stresses, etc. Structural designers aim

to ensure their safety and serviceability under all expected loads during their service life. Being design-oriented, research investigations in concrete engineering attempt to study the effect of all the above factors and thus follow the empirical-computational methodology. As expected, such is also the case of researches on inelastic stability of concrete columns and beam-columns. Various approximate computational methods containing many empirical coefficients have been proposed for quantifying the buckling loads or effective lengths for different slenderness ratios, end conditions, concrete and steel grades, amounts of reinforcement, etc. After extensive empirical validation, design methodologies are codified. The scope of these empirical investigations is restricted to only their static stability under conservative forces. Extensive literature survey has revealed the absence of any investigations into dynamic stability of concrete beam-columns under conservative and nonconservative forces. There exists diversity of views amongst researches regarding the choice of static and kinematic reference axes.

It is well known that concrete structures are cracked at discrete locations even under service loads. The pattern and extent of cracks depends upon load history. Crack formation is an inelastic process whereas closing and reopening of the extant cracks results in nonlinear elastic behaviour. To recapitulate, concrete engineering scientists study inelastic instability while the scope of the classical theory of elastic stability is restricted to physically linear elastic beam-columns undergoing finite lateral displacements. Thus, the stability of physical nonlinear elastic concrete beam-columns has not yet been studied. Fully cracked nonlinear elastic concrete structures have earlier been modelled as first order homogeneous mechanical systems. The nonlinear dynamic behaviour of fully cracked SDOF and two-DOF concrete beams has also been explored.

The objective of present Thesis is to investigate the static and dynamic elastic stability of fully cracked concrete beam-columns subjected to conservative and nonconservative forces. Cracks closed under compression are indistinguishable from uncracked concrete. The chosen object of study is a vertical flanged concrete cantilever under the action an axial force and a lateral force acting at the free end carrying lumped mass at that point. These beam-columns can experience finite lateral displacements but only small slope. Following an analytical-computational methodology, constitutive equations in the form of force-displacement relations are derived. Here, the eccentricity of loading and section moment of inertia are defined in reference to the instantaneous elastic centroidal axis of the concrete beam-columns while the lateral displacements, slopes and curvatures are referred to the elastic centroidal axis of the uncracked beam-column in the passive state. Only lateral displacements are considered which result only in SDOF physically nonlinear elastic dynamical systems. Using the derived expression for lateral stiffness, and so the damping coefficient, the required equations of motion are derived. Their static and dynamic response is then computed for different loading histories.

For these elastic structures, there are two sources of nonlinearity, viz, physical and geometrical nonlinearity but only geometrically nonlinear structures are expected to exhibit loss of static and dynamic stability. Static stability of concrete beam-columns under constant lateral forces and conservative axial force is investigated. Two critical values of both the axial and lateral loads are identified. Critical axial load for concrete beam-columns cracked at all the sections is lesser than that for the uncracked beam-columns. For constant lateral force smaller than its lower critical value, the concrete beam-columns exhibit brittle buckling mode characterised by peak axial load at small lateral displacements. Higher lateral forces lesser than the second critical value introduce alternate stable and unstable domains with increase in axial force. The lateral stiffness is predicted to vanish when the axial loads reach the critical values and when the limiting displacement is reached for axial load exceeding its lower critical value. The load-space is partitioned into statically stable and unstable regions for different load paths. Eccentrically loaded pinned-pinned flanged concrete columns have also been predicted to exhibit similar buckling response. Effect of eccentricity of loading, amount of longitudinal reinforcement and concrete grade on the buckling behaviour has been investigated. It has been found that lightly reinforced columns made of higher grade of concrete and loaded at smaller eccentricity exhibit brittle buckling behaviour.

Using the derived expression for the lateral stiffness under constant axial force, their elastodynamic stability of concrete beam-columns under conservative loads is investigated. As expected, the instantaneous values of the stiffness and the damping coefficients of the lumped-mass underdamped SDOF nonlinear structures are found to depend upon the vibration amplitude. The natural frequency has been found to vanish at the two critical axial loads. For axial load exceeding the lower critical value, the concrete beam-columns in the second equilibrium state are shown to exhibit loss of dynamic stability by divergence. Depending upon the initial conditions, the phase plane has been partitioned into dynamically stable and unstable regions. Under harmonic excitations, the nonlinear dynamical systems exhibit subharmonic resonances and jump phenomena. Loss of dynamic stability has been predicted for some ranges of damping ratio as well as of peak sinusoidal force and forcing frequency. Sensitivity of dynamic stability to the initial conditions and the sense of the peak sinusoidal force have also been predicted. Similar but stable dynamical behaviour is also exhibited by concrete beam-columns undergoing small lateral displacements.

Two measures – damping ratio and damping coefficient – of structural damping have been employed. Critical loads and displacements are not affected by level or measure of structural damping. Dynamic instability by divergence at all levels of damping occurs for the case of statically unstable load sets. In particular cases, higher damping has been predicted to destabilize even these conservative structures. Effects of initial conditions and higher damping on the inelastic stability for equivalent viscous damping, passive stability control and creep-like buckling of concrete beam-columns have been delineated.

A dynamic instability, called parametric resonance, is exhibited by undamped elastic beam-columns when under the action of pulsating axial force. The scope of the existing theory of parametric resonance is restricted to physically linear beam-columns undergoing finite lateral displacements. In this Thesis, the dynamic behaviour of physically nonlinear elastic cracked concrete beam-columns under pulsating axial force and constant lateral force is investigated. The constitutive equations derived above in the form of force-displacement relations are employed here to formulate equations of motion of the SDOF cantilever with mass lumped at its free end. The expected phenomenon of parametric resonance is exhibited in the form of regular subharmonic resonance at about the frequency ratio of two. Resonance peaks broaden with increase in pulsating force. Like damping, physical nonlinearity is also predicted to stabilize the dynamic response at resonance frequencies. In some particular statically unstable conditions, the loss of dynamic stability is shown to occur by divergence. Unexpectedly, similar phenomenon of parametric resonance is exhibited by these physically nonlinear beam-columns undergoing even small lateral displacements.

An axial follower force acting on the free end of a beam-column is known to remain tangential to its elastica at that point. Elastic beam-columns exhibit infinitely high buckling resistance to static compressive follower load. Loss of their dynamic stability is known to occur at critical follower load by flutter characterized by vanishing lateral displacement and infinitely high natural frequency. Classical theory of stability under follower forces deals with physically linear nonconservative beam-columns. Here, elastic analytical expressions for the lateral displacement and lateral stiffness of nonlinear concrete beam-columns are derived. Using these expressions, the stability of physically nonlinear elastic flanged concrete beam-columns under the action of a follower compressive axial force and a lateral force is investigated. Only one out of the possible three equilibrium states in the range of the lowest two flutter loads has been found to be dynamically stable. The variation of lateral stiffness with increase in follower force is much more complex than in the case of physically linear beam-columns. Cracking is predicted lower the flutter loads. Such flutter in cracked beam-columns occurs only when the applied lateral loads exceed certain critical value. Load space is partitioned into dynamically stable and unstable regions. A quasi-static instability like snap-through is predicted when cracking axial load exceeds the critical conservative load for uncracked beam-columns.

Well-designed elevated concrete water tanks with shaft-type staging have been observed to exhibit poor seismic performance. Their elastic analysis based on gross lateral stiffness ignores the effect of cracks and reinforcement. The nonlinear elastic vibration response of water tanks with circular shaft-type staging initially cracked in flexural tension is investigated here. These structures with realistic details have been modelled as vertical flanged concrete columns supporting weight of the water container at their free end. Subharmonic resonances in addition to the fundamental resonance, discontinuities in the frequency and time domain response, extreme sensitivity to initial conditions and forcing frequencies, and even chaotic vibration response have been predicted. The effect of presence of cracking on the finite amplitude lateral vibrations has also been investigated.

Also, these concrete structures are shown to exhibit the dynamic instability by parametric resonance under pulsating axial force. Beam-columns with extremely small or nil reinforcement are interpreted to resemble overhead water tanks with no-tension masonry shafts. Validity of the proposed theory to sway-frame columns and to machine-supporting framed foundations has also been explored. Ferrocement or reinforced concrete structures with distributed reinforcement can be modelled as bimodular solids. The proposed theory is shown to be valid for such bimodular flanged beam-columns.

In this Thesis, the classical theory of elastic stability has been extended to physically nonlinear elastic beam-columns. The effect of physical nonlinearity on static and dynamic stability under conservative loads as well on the dynamic instabilities like flutter under follower loads and parametric resonance under pulsating axial force has been quantified. The proposed theory of elastic stability has potential application for the concrete beam-columns under service loads. Rational methodology has been adopted here in preference to the empirical methodology currently popular amongst concrete engineering scientists. It is claimed that the theory of elastic stability of physically nonlinear beam-columns proposed in this Thesis is a significant contribution to the theory of elastic stability in general and to the theory of concrete beam-columns in particular. Due to lack of relevant experimental data, the theoretical predictions have not been fully validated. Further, experimental, analytical and computational research areas are suggested.

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