

Algorithmic Study of Some Graph Problems
On
Certain Special Graph Classes

By

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Certificate

This is to certify that the thesis entitled “**Algorithmic Study of Some Graph Problems On Certain Special Graph Classes**” submitted by “**Mr. Dina-bandhu Pradhan**” to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

Graphs theory plays an important role in modeling problems arising in several practical applications. However, most of the computational graph theoretic problems are NP-hard for general graphs. Graphs used in models possess some special structures. Therefore, the study of NP-hard graph problems restricted to special graph classes, has generated a lot of interests and is an active area of research in algorithmic graph theory.

In this thesis, we study the algorithmic aspects of four important graph problems, namely (i) paired-domination in graphs, (ii) restrained domination in graphs, (iii) maximal acyclic matching in graphs and (iv) locally connected spanning tree in graphs. All of these four problems are known to be NP-hard for general graphs.

A set $D \subseteq V$ of a graph $G = (V, E)$ is a *paired-dominating set* of G if every vertex in $V \setminus D$ has a neighbor in D and $G[D]$, the induced subgraph by G on D , has a perfect matching. Given a graph $G = (V, E)$ and a positive integer k , the *paired-domination problem* is to decide whether G has a paired-dominating set of cardinality at most k . The paired-domination problem is known to be NP-complete for bipartite graphs. In this thesis, we propose linear time algorithms for computing a minimum paired-dominating set in convex bipartite graphs and chordal bipartite graphs. We also show that the paired-domination problem remains NP-complete for perfect elimination bipartite graphs.

A set $D \subseteq V$ of a graph $G = (V, E)$ is a *restrained dominating set* of G if every vertex in $V \setminus D$ is adjacent to a vertex in D as well as to another vertex in $V \setminus D$. Given a graph G and a positive integer k , the *restrained domination problem* is to check whether G has a restrained dominating set of size at most k . The restrained domination problem is known to be NP-complete for chordal graphs. In this thesis, we show that the restrained domination problem remains NP-complete for split graphs and undirected path graphs. We also propose a linear time algorithm for computing a minimum restrained dominating set in proper interval graphs.

A matching M of a graph G is an *acyclic matching* of G if $G[V(M)]$ is acyclic. An acyclic matching M of G is called a *maximal acyclic matching* if M is not a proper subset of any other acyclic matching of G . A maximum (minimum) cardinality maximal acyclic matching M of G is called a *maximum (minimum) maximal acyclic matching* of G . Given a graph $G = (V, E)$ and a positive integer k , the *maximum (minimum) maximal acyclic matching* problem is to decide whether the graph G has a maximal acyclic matching M of cardinality at least k (at most k). The maximum maximal acyclic matching problem is known to be NP-complete for general graphs. In this thesis, we propose linear time algorithms for computing a maximum maximal acyclic matching in chain graphs, bipartite permutation graphs and proper interval graphs. We also show that the maximum maximal acyclic matching problem is NP-complete for perfect elimination bipartite graphs. We obtain some hardness and approximation results of the problem of finding a maximum maximal acyclic matching. We also show that the minimum maximal acyclic matching problem is NP-complete for bipartite graphs.

A spanning tree T of a graph $G = (V, E)$ is called a *locally connected spanning tree* if the set of all neighbors of v in T induces a connected subgraph in G for every $v \in V(G)$. The locally connected spanning tree recognition problem is to recognize whether a given graph G admits a locally connected spanning tree. The locally connected spanning tree construction problem is to construct a locally connected

spanning tree of a given graph that is known to admit a locally connected spanning tree. The locally connected spanning tree recognition problem is known to be NP-complete for chordal graphs. In this thesis, we propose linear time algorithms for the locally spanning tree recognition and construction problems for cographs, complements of bipartite graphs and doubly chordal graphs. A Hamiltonian path of a graph G that is a locally connected spanning tree of G is called as a Hamiltonian locally connected spanning tree. In this thesis, we show that the problem of deciding whether a graph G has a Hamiltonian locally connected spanning tree is NP-complete for undirected path graphs and directed path graphs. We also propose a simple linear time algorithm for the recognition and construction problems of Hamiltonian locally connected spanning tree in proper interval graphs.

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