

ANALYSIS OF HYDRODYNAMIC PROBLEMS  
USING  
FINITE AND INFINITE ELEMENTS

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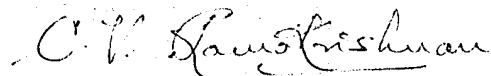
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## ABSTRACT

With rapid development of offshore technology and naval structural engineering in recent years hydrodynamic problems have received a considerable attention. In the present investigation infinite elements coupled with finite elements have been used to solve a variety of problems raised by unbounded domain. It has the advantage of producing symmetric and banded equation systems. Variations in depth and geometry changes can be dealt accurately. The radiation condition has been accommodated easily by the infinite elements.

Four computer programs have been developed for this purpose. Wave load interaction phenomenon in the context of large vertical cylinders extending from the seabed and piercing the free surface has been studied. The wave forces due to interaction effects are compared with experimentally measured forces and analytical results. Satisfactory prediction of the rigid body motions of a surface ship is required for evaluation of hydrodynamic forces apart from hydrostatic forces. Two dimensional evaluation of hydrodynamic forces for different hull shapes has been carried out using finite and infinite elements for different depths and shapes. Hydrodynamic forces for infinite depth has also been evaluated using infinite elements.

One of the more common geometries employed in offshore operations is, after the horizontal slender body, the case of vertical axisymmetry. Both rigid body oscillation and structural vibrations can be relevant depending upon the relationship between wave frequency and lowest natural frequency of vibration of the structure.

A numerical technique for obtaining the hydrodynamic loads on a wide class of these structures, using finite and infinite elements has been developed. The method has been extended for the evaluation of surface elevation of an axisymmetric wavemaker.

The study of transient wave problems in hydrodynamics has been conducted. The use of infinite elements for representing the infinite domain in time dependent problems and the estimation of disturbance in the proximity of the source has been obtained. Development of computer programs for the above objectives has been explained in brief.

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