SOME STUDIES ON CHARACTERIZATION AND MODELING OF STOCHASTIC PROCESSES IN THE MULTISCALE FRAMEWORK

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CERTIFICATE

This is to certify that the thesis entitled **"Some Studies on Characterization and Modeling of Stochastic Processes"** being submitted by **Mr. Brejesh Lall** to the Department of Electrical Engineering, Indian Institute of Technology, Delhi for the award of the degree of **Doctor of Philosophy** is a record of bonafide research work carried out by him under our supervision and guidance. The results contained in this thesis have not been submitted to any other Institute for the award of any degree or diploma

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Abstract

This thesis is an attempt to obtain a multiscale representation for stochastic processes. This thesis, besides obtaining the multiscale model, derives an alternative characterization for the filter bank. This alternative characterization is an illustrative representation of the transformations taking place in the signal as it propagates through the filter bank. The derivation of this characterization also leads to certain interesting relationships between the input and output correlations and spectrums of certain multirate filters (for WSCS inputs), which are elements of the filter bank.

The thesis builds a framework for the proposed multiscale model, by obtaining an illustrative characterization of the filter bank. For this an alternative representation for wide sense cyclostationary (WSCS) processes is -obtained. This representation is important, as it is well known that the output of a filter bank, for a wide sense stationary (WSS) or a WSCS input, is WSCS. This alternative characterization of WSCS process is a time frequency representation of the WSCS process. The WSCS process, with period M, can be characterized using M distinct correlation sequences. The Fourier Transform of these correlation sequences gives M distinct spectrums, which can be used to represent the WSCS process, with period M.

Using this alternative characterization of WSCS processes, we obtain the input-output relationships of certain multirate filters (for WSCS inputs). In particular those multirate filters that constitute the filter bank, have been discussed in detail. Once these relationships are available, an alternative characterization of the filter bank is obtained. This characterization explicitly brings out the transfonnations that take place in a signal as it propagates through a filter bank. Relationships between the input and the output

correlation sequences and the spectrums are obtained in terms of the filter bank elements' transfer functions.

The proposed characterization is used to obtain the conditions, on the filter bank elements, that lead to perfect reconstruction by the filter bank. These conditions are obtained by matching the second order statistics of the input and the output of the filter bank. The output spectrums are obtained in terms of the input spectrums and the filter bank elements' transfer functions. These spectrums are equated to the input spectrums to obtain the conditions for perfect reconstruction.

Besides conditions for perfect reconstruction, conditions (in terms of the filter bank elements' transfer functions) that lead to the output of the filter bank being stationary (for WSS input) have also been derived. Also, the constraints that lead to the output of certain other multirate filter configurations being stationary have also been derived. In particular, these conditions have been obtained for the synthesis stage of a two-band filter bank and also for the proposed realization of the multiscale model presented in this thesis.

This thesis proposes a multiscale model for representation of stationary and cyclostationary processes. The proposed model extracts the information from the immediate coarser scale, and adds innovations to it to obtain the finer scale representation of the stochastic process. The coarser scale version samples are passed through an upsampler, and then through an MA filter to extract the information of the finer scale version available at this scale. To this- information innovations are added. These innovations are obtained by passing cyclostationary white noise through an AR filter to provide appropriate coloring to it. The mapping from the finer scale to the immediate coarser scale is done using an assumed blurring model. The proposed model is extended to the two-dimensional random field case, where both the separable and the non-separable models have been discussed.

For a representation to be useful, regeneration of the stochastic process from the model parameters should be possible. Also, given a realization of the process, or its statistics, model parameters should be obtainable. In this thesis, we have obtained solutions for the model parameters for the given statistics case as well as the given data case. The solutions are developed for one-dimensional model and also for the two-dimensional model case. In the two-dimensional model, solutions for the parameters for both the separable and the non-separable case have been derived.

The simulation results for the proposed model have also been given in this thesis. The various processes that can be generated using this multiscale model are given. The estimation techniques proposed in this thesis are used to model a given stochastic process. These parameters are then used to regenerate the stochastic process. The second order statistics of the two processes, the original and the regenerated processes, is compared using a number of simulations. The two statistics match well, as shown in this thesis.

Finally, the thesis contains some spin-off resulting from the work done during the course of the thesis. In particular, another multiscale modeling scheme is proposed. In this other multiscale model, the immediate coarser scale version of the process is simply the downsampled version of the finer scale version. Here again the finer scale version is generated using the coarser scale version samples, and then adding innovations, which are samples of a white noise process. Here only those samples of the immediate coarser scale version are used for generating the finer scale version, which are less than or equal to a certain distance from the sample being generated. The notion of distance here is that of the one in a binary tree, since the binary tree has been used as the index set (for the stochastic process and its finer scale versions) in this multiscale model.

Another spin-off discussed in this thesis is the wavelet matched to the statistics of the stochastic process. The parameters defining the blurring mechanism of the proposed multiscale model represent the two-scale relation and are, therefore, the basis for obtaining the wavelet for representing the given stochastic processes. These parameters are matched to the second order statistics of the stochastic process to obtain the statistically matched wavelet. The thesis concludes with a brief discussion on the possible future directions for research, and potential problems that emanate from the work done in this thesis.

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