

Geohazard Dynamics: SPH-Driven Engineering Approach

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May 2025

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Submitted

in fulfilment of the requirements of the degree of

Doctor of Philosophy

to the



Indian Institute of Technology Delhi

Hauz Khas, New Delhi-110016, India

May 2025

Certificate

This is to certify that the thesis entitled “**Geohazard Dynamics: SPH-Driven Engineering Approach**”, being submitted by **Mr Sourabh Mhaski** to the Indian Institute of Technology Delhi, is a record of bona fide research work carried out by him under my supervision and guidance. In my opinion, the thesis work has reached the standard that fulfils the requirements for the **Doctor of Philosophy** degree. The research report and results presented in this thesis have not been submitted, in part or full, to any University or Institute for the award of any degree or diploma.

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Acknowledgements

I express my heartfelt gratitude to my supervisor, Prof. G. V. Ramana, for his invaluable guidance, unwavering support, and insightful feedback throughout this journey. His profound knowledge, constant encouragement, and patience have been instrumental in shaping the direction and quality of this research. I am also grateful to the members of my research committee, Prof. D. R. Kaushal, Prof. Puneet Mahajan, and Prof. N. M. Anoop Krishnan, for their suggestions and constructive feedback.

I would like to acknowledge my friends and colleagues, Dr Deepesh Bansal, Mr Avinash Sajwan, Mr Deepak Haritwal, Mr Pranjal Singh, and Mr G. Shiva Shankar Reddy, for their support and engaging discussions throughout this journey. Their willingness to share knowledge and exchange ideas has been a constant source of inspiration and motivation. Finally, I extend my sincere gratitude to my family for their patience and belief in me throughout this endeavour.

Sourabh Mhaski

Abstract

Geohazards such as landslides, erosion, sedimentation, liquefaction, ground subsidence, ground collapse, and subsurface contamination are a global concern and cause loss of human lives and extensive economic and environmental damage. The increasing intensity and frequency of these events, driven by factors such as climate change, rapid urbanisation, and population growth, highlight the significance and the pressing need for effective geohazard mitigation strategies. Developing an engineering-based understanding of geohazards is crucial for devising practical solutions to reduce risk, improve the resilience of built infrastructure, and ensure the safety and sustainability of communities.

The present research focuses on developing an accurate and reliable numerical modelling framework for analysing geohazards from a geotechnical engineering perspective. Geohazards are complex, multiphysics phenomena, generally driven by interactions between geomaterials (such as soil, coal ash, mine tailings, Municipal Solid Waste (MSW), etc.) and fluids like water, and often involve large material deformations. To effectively model these characteristics, a numerical framework based on the Smoothed Particle Hydrodynamics (SPH) method for modelling geohazards as coupled hydromechanical processes in multiphase systems is presented. Based on a state-of-the-art review of geohazard modelling using SPH, the following challenging and practically significant problems were identified for this research:

- **Uncertainty-based stability analysis of MSW dumpsites:** MSW dumpsites are marginally stable structures constructed by non-engineered waste disposal in several developing countries. Instability of MSW dumpsites is often triggered by rainfall and high leachate levels. In addition to the lack of knowledge of in-situ conditions at dumpsites, further uncertainty in the stability analysis is introduced due to the highly heterogeneous nature of the material (MSW). As a result, a probabilistic risk assessment approach is essential.
- **Examination of the effect of internal erosion on coal ash ponds:** Coal-based power generation will continue to be the backbone of energy supply until renewable energy sources become mainstream. Hence, the management of large quantities of coal ash generated as a by-product is a global concern. Coal ash is commonly disposed of in ash ponds and is prone to internal erosion because of its low specific gravity. Thus, design and analysis techniques incorporating internal erosion are essential for constructing resilient ash ponds.

- **Earthquake-induced liquefaction hazard assessment:** Earthquake-induced soil liquefaction poses significant risks to the stability of geotechnical structures worldwide. A comprehensive liquefaction hazard assessment involves understanding the critical conditions leading to the triggering of liquefaction, as well as the post-failure large deformation behaviour. A numerical framework capable of handling both aspects is necessary for the design of resilient infrastructure.
- **Coupled flow-solute transport model for subsurface contamination:** Subsurface contamination is often hazardous to human health and the environment. Understanding the coupled process of flow of water and contaminant (solute) transport in porous media is essential for assessing the extent, designing mitigation measures, and enhancing resilience against subsurface contamination.

The above-described geohazards are distinct in their mechanisms and effects, yet governed by similar underlying hydromechanical interactions. In this research, the SPH-based numerical framework is developed in alignment with this principle. Specifically,

- A two-phase Random SPH (RSPH) approach is developed for modelling large deformation MSW mass flows, incorporating spatial variability of material properties and pore water pressure using the random field theory. The variability of MSW properties is characterised for uncertainty modelling based on the analysis of 584 laboratory tests on MSW from 28 countries. The proposed approach is adopted for risk assessment of three operational dumpsites, namely, (i) Ghazipur dumpsite, Delhi, India; (ii) Koshe dumpsite, Addis Ababa, Ethiopia; and (iii) Hulene dumpsite, Maputo, Mozambique. Investigation of the uncontrolled and controlled surface infiltration scenarios indicates that providing a cover system over the waste dump is an effective risk minimisation technique, resulting in over 30% reduction in the runout distance and over 40% in the maximum kinetic energy of the mass flow. Further, deterministic analysis with homogeneous material properties is found to underestimate the vulnerability to failure, highlighting the significance of probabilistic approaches in identifying the high-risk zones around an MSW dumpsite.
- The effect of internal erosion on the stability of ash ponds is investigated using a five-phase SPH model. The approach is utilised to analyse an operational ash pond from southeast India, where instability due to internal erosion was observed. The results indicate that the amount of erodible fly ash significantly influences the initiation of instability and the subsequent mass flow, with nearly 100% reduction in mass flow as the fly ash content is reduced from 80% to 20% by weight. The significance of appropriate coal ash slurry disposal practices and field monitoring for

averting such failures is also highlighted. Further, replacing the erodible coal ash with a non-erodible material is validated as a feasible failure mitigation measure.

- A novel SPH framework for earthquake-induced liquefaction hazard assessment of geotechnical structures using SPH is developed. The coupled flow-deformation behaviour of soils subjected to cyclic loading is described using the PM4Sand model implemented in a three-phase SPH model. A staggered discretisation scheme based on the stress particle SPH approach is adopted to minimise numerical inaccuracies caused by zero-energy modes and tensile instability. Further, non-reflecting boundary conditions for seismic analysis of semi-infinite soil domains using the SPH method are developed. The numerical framework is employed for the analysis of the cyclic direct simple shear test, seismic analysis of a level ground site, and liquefaction-induced failure of the Lower San Fernando Dam. Satisfactory agreement for liquefaction triggering and post-failure behaviour demonstrates that the framework can be utilised to assess the effect of seismic loading on field-scale geotechnical structures.
- An SPH-based numerical framework is developed for modelling coupled flow-solute transport in three-phase, non-deformable porous media. A generalised theoretical model describing the coupled process is arrived at based on the continuum mixture theory. The developed numerical framework is validated against closed-form solutions of one- and two-dimensional solute transport in porous media, the analytical solution of the Henry problem, and a laboratory-scale experiment of saltwater intrusion involving density-driven flow. Satisfactory agreement with the analytical, numerical, and experimental results indicates that the proposed framework can be adopted for the coupled analysis of flow of water and solute transport in porous media for subsurface contamination studies.

The diverse range of complex, multiphysics, field-scale problems addressed in this thesis highlights the versatility and robustness of the developed SPH-based numerical framework. This study advances the state of knowledge of the SPH method for geohazard modelling and is expected to serve as a basis for future developments and applications of SPH in this field.

सारांश

भूस्खलन, अपरदन, अवसादन, द्रवीकरण, भूमि धंसाव, भूस्खलन और उपसतही संदूषण जैसे भू-आपदाएँ एक वैश्विक चिंता का विषय हैं और इनके कारण मानव जीवन की हानि और व्यापक आर्थिक एवं पर्यावरणीय क्षति होती है। जलवायु परिवर्तन, तीव्र शहरीकरण और जनसंख्या वृद्धि जैसे कारकों से प्रेरित इन घटनाओं की बढ़ती तीव्रता और आवृत्ति, प्रभावी भू-आपदा न्यूनीकरण रणनीतियों के महत्व और अत्यावश्यकता को उजागर करती है। जोखिम को कम करने, निर्मित बुनियादी ढांचे के लचीलेपन में सुधार करने और समुदायों की सुरक्षा और स्थिरता सुनिश्चित करने के लिए भू-आपदाओं की इंजीनियरिंग-आधारित समझ विकसित करना महत्वपूर्ण है।

वर्तमान शोध भू-तकनीकी इंजीनियरिंग के दृष्टिकोण से भू-आपदाओं के विश्लेषण के लिए एक सटीक और विश्वसनीय संख्यात्मक मॉडलिंग ढांचा विकसित करने पर केंद्रित है। भू-आपदाएँ जटिल, बहुभौतिकीय घटनाएँ हैं, जो आम तौर पर भू-सामग्री (जैसे मिट्टी, कोयला राख, खदान अपशिष्ट, नगरपालिका ठोस अपशिष्ट (एमएसडब्ल्यू), आदि) और पानी जैसे तरल पदार्थों के बीच परस्पर क्रिया द्वारा संचालित होती हैं, और अक्सर इनमें बड़े पैमाने पर सामग्री विरूपण शामिल होते हैं। इन विशेषताओं को प्रभावी ढंग से मॉडल करने के लिए, बहु-चरण प्रणालियों में युग्मित हाइड्रोमैकेनिकल प्रक्रियाओं के रूप में भू-आपदाओं के मॉडलिंग के लिए स्मूथेड पार्टिकल हाइड्रोडायनामिक्स (एसपीएच) विधि पर आधारित एक संख्यात्मक ढांचा प्रस्तुत किया गया है। एसपीएच का उपयोग करके भू-आपदा मॉडलिंग की अत्याधुनिक समीक्षा के आधार पर, इस शोध के लिए निम्नलिखित चुनौतीपूर्ण और व्यावहारिक रूप से महत्वपूर्ण समस्याओं की पहचान की गई:

- **एमएसडब्ल्यू कचरा स्थलों का अनिश्चितता-आधारित स्थिरता विश्लेषण:** एमएसडब्ल्यू कचरा स्थल कई विकासशील देशों में गैर-इंजीनियर अपशिष्ट निपटान द्वारा निर्मित सीमांत रूप से स्थिर संरचनाएं हैं। एमएसडब्ल्यू कचरा स्थलों की अस्थिरता अक्सर वर्षा और उच्च लीचेट स्तरों से शुरू होती है। कचरा स्थलों पर यथास्थान स्थितियों के ज्ञान की कमी के अलावा, सामग्री (एमएसडब्ल्यू) की अत्यधिक विषम प्रकृति के कारण स्थिरता विश्लेषण में और अनिश्चितता आ जाती है। नतीजतन, एक संभाव्य जोखिम मूल्यांकन दृष्टिकोण आवश्यक है।
- **कोयला राख तालाबों पर आंतरिक कटाव के प्रभाव की जांच:** जब तक नवीकरणीय ऊर्जा स्रोत मुख्यधारा में नहीं आ जाते, तब तक कोयला आधारित बिजली उत्पादन ऊर्जा आपूर्ति की रीढ़ बना रहेगा। इसलिए, उप-उत्पाद के रूप में उत्पन्न बड़ी मात्रा में कोयला राख का प्रबंधन एक वैश्विक चिंता का विषय है। कोयला राख को आमतौर पर राख तालाबों में निपटाया जाता है और कम विशिष्ट गुरुत्व के कारण यह आंतरिक कटाव के प्रति संवेदनशील होती है। इस प्रकार,

लचीले राख तालाबों के निर्माण के लिए आंतरिक कटाव को शामिल करने वाली डिजाइन और विश्लेषण तकनीकें आवश्यक हैं।

- **भूकंप-जनित द्रवीकरण खतरा आकलन:** भूकंप-जनित मृदा द्रवीकरण विश्व स्तर पर भू-तकनीकी संरचनाओं की स्थिरता के लिए महत्वपूर्ण जोखिम पैदा करता है। एक व्यापक द्रवीकरण खतरा आकलन में द्रवीकरण को शुरू करने वाली महत्वपूर्ण स्थितियों के साथ-साथ विफलता के बाद के बड़े विरूपण व्यवहार को समझना शामिल है। लचीले बुनियादी ढांचे के डिजाइन के लिए दोनों पहलुओं को संभालने में सक्षम एक संख्यात्मक ढांचे की आवश्यकता है।
- **उपसतही संदूषण के लिए युग्मित प्रवाह-विलेय परिवहन मॉडल:** उपसतही संदूषण अक्सर मानव स्वास्थ्य और पर्यावरण के लिए खतरनाक होता है। छिद्रयुक्त माध्यमों में पानी के प्रवाह और संदूषक (विलेय) परिवहन की युग्मित प्रक्रिया को समझना, सीमा का आकलन करने, न्यूनीकरण उपायों को डिजाइन करने और उपसतही संदूषण के खिलाफ लचीलापन बढ़ाने के लिए आवश्यक है।

ऊपर वर्णित भू-आपदाएँ अपनी क्रियाविधि और प्रभावों में भिन्न हैं, फिर भी समान अंतर्निहित हाइड्रोमैकेनिकल अंतःक्रियाओं द्वारा शासित होती हैं। इस शोध में, एसपीएच-आधारित संख्यात्मक ढाँचा इसी सिद्धांत के अनुरूप विकसित किया गया है। विशेष रूप से,

- बड़े विरूपण वाले एमएसडब्ल्यू द्रव्यमान प्रवाह के मॉडलिंग के लिए एक द्वि-चरण यादृच्छिक एसपीएच (आरएसपीएच) दृष्टिकोण विकसित किया गया है, जिसमें यादृच्छिक क्षेत्र सिद्धांत का उपयोग करके सामग्री गुणों और छिद्र जल दबाव की स्थानिक परिवर्तनशीलता को शामिल किया गया है। 28 देशों के एमएसडब्ल्यू पर किए गए 584 प्रयोगशाला परीक्षणों के विश्लेषण के आधार पर अनिश्चितता मॉडलिंग के लिए एमएसडब्ल्यू गुणों की परिवर्तनशीलता का वर्णन किया गया है। प्रस्तावित दृष्टिकोण को तीन चालू कचरा स्थलों, अर्थात् (i) गाजीपुर कचरा स्थल, दिल्ली, भारत; (ii) कोशे कचरा स्थल, अदीस अबाबा, इथियोपिया; और (iii) हुलेन कचरा स्थल, मापुटो, मोजाम्बिक के जोखिम मूल्यांकन के लिए अपनाया गया है। अनियंत्रित और नियंत्रित सतह अंतःस्यंदन परिदृश्यों की जांच से पता चलता है कि अपशिष्ट डंप पर एक आवरण प्रणाली प्रदान करना एक प्रभावी जोखिम न्यूनीकरण तकनीक है, जिसके परिणामस्वरूप रनआउट दूरी में 30% से अधिक और द्रव्यमान प्रवाह की अधिकतम गतिज ऊर्जा में 40% से अधिक की कमी आती है। इसके अलावा, सजातीय सामग्री गुणों के साथ नियतात्मक विश्लेषण विफलता की भेद्यता को कम आंकता हुआ पाया गया है, जो एमएसडब्ल्यू कचरा स्थल के आसपास उच्च जोखिम वाले क्षेत्रों की पहचान करने में संभाव्य दृष्टिकोणों के महत्व पर प्रकाश डालता है।
- राख तालाबों की स्थिरता पर आंतरिक कटाव के प्रभाव की जांच एक पांच-चरण एसपीएच मॉडल का उपयोग करके की जाती है। इस दृष्टिकोण का उपयोग दक्षिण-पूर्व भारत के एक चालू राख तालाब का विश्लेषण करने के लिए किया जाता है, जहाँ आंतरिक कटाव के कारण अस्थिरता

देखी गई थी। परिणाम बताते हैं कि अपरदनशील फ्लाइ ऐश की मात्रा अस्थिरता की शुरुआत और उसके बाद के द्रव्यमान प्रवाह को महत्वपूर्ण रूप से प्रभावित करती है, फ्लाइ ऐश की मात्रा को वजन के हिसाब से 80% से 20% तक कम करने पर द्रव्यमान प्रवाह में लगभग 100% की कमी आती है। इस तरह की विफलताओं से बचने के लिए उचित कोयला राख घोल निपटान प्रथाओं और क्षेत्र निगरानी के महत्व पर भी प्रकाश डाला गया है। इसके अलावा, अपरदनशील कोयला राख को गैर-अपरदनशील सामग्री से बदलना एक व्यवहार्य विफलता शमन उपाय के रूप में मान्य किया गया है।

- एसपीएच का उपयोग करके भू-तकनीकी संरचनाओं के भूकंप-जनित द्रवीकरण खतरा आकलन के लिए एक नया एसपीएच ढांचा विकसित किया गया है। चक्रीय लोडिंग के अधीन मिट्टी के युग्मित प्रवाह-विरूपण व्यवहार को तीन-चरण एसपीएच मॉडल में कार्यान्वित PM4Sand मॉडल का उपयोग करके वर्णित किया गया है। शून्य-ऊर्जा मोड और तन्य अस्थिरता के कारण होने वाली संख्यात्मक अशुद्धियों को कम करने के लिए तनाव कण एसपीएच दृष्टिकोण के आधार पर विवेकीकरण योजना अपनाई गई है। इसके अलावा, एसपीएच विधि का उपयोग करके अर्ध-अनंत मिट्टी डोमेन के भूकंपीय विश्लेषण के लिए गैर-परावर्तक सीमा स्थितियाँ विकसित की गई हैं। संख्यात्मक ढांचे का उपयोग चक्रीय प्रत्यक्ष अपरूपण परीक्षण के विश्लेषण, एक समतल भूभाग स्थल के भूकंपीय विश्लेषण और लोअर सैन फर्नांडो बांध की द्रवीकरण-प्रेरित विफलता के लिए किया जाता है। द्रवीकरण के उत्प्रेरण और विफलता के बाद के व्यवहार के लिए संतोषजनक सहमति दर्शाती है कि इस ढांचे का उपयोग क्षेत्र-स्तरीय भू-तकनीकी संरचनाओं पर भूकंपीय लोडिंग के प्रभाव का आकलन करने के लिए किया जा सकता है।
- तीन-चरण, गैर-विरूपणशील छिद्रयुक्त माध्यमों में युग्मित प्रवाह-विलेय परिवहन के मॉडलिंग के लिए एक एसपीएच-आधारित संख्यात्मक ढांचा विकसित किया गया है। सातत्य मिश्रण सिद्धांत के आधार पर युग्मित प्रक्रिया का वर्णन करने वाला एक सामान्यीकृत सैद्धांतिक मॉडल तैयार किया गया है। विकसित संख्यात्मक ढांचे को छिद्रयुक्त माध्यमों में एक- और द्वि-आयामी विलेय परिवहन के क्लोज्ड-फॉर्म समाधानों, हेनरी समस्या के विश्लेषणात्मक समाधान और घनत्व-चालित प्रवाह से जुड़े खारे पानी के अंतःकरण के एक प्रयोगशाला-स्तरीय प्रयोग के विरुद्ध मान्य किया गया है। विश्लेषणात्मक, संख्यात्मक और प्रायोगिक परिणामों के साथ संतोषजनक सहमति इंगित करती है कि प्रस्तावित ढांचे को उपसतही संदूषण अध्ययनों के लिए छिद्रयुक्त माध्यमों में पानी के प्रवाह और विलेय परिवहन के युग्मित विश्लेषण के लिए अपनाया जा सकता है।

इस शोध प्रबंध में संबोधित जटिल, बहुभौतिकीय, क्षेत्र-स्तरीय समस्याओं की विविध श्रेणी विकसित एसपीएच-आधारित संख्यात्मक ढांचे की बहुमुखी प्रतिभा और मजबूती पर प्रकाश डालती है। यह अध्ययन भू-आपदा मॉडलिंग के लिए एसपीएच विधि के ज्ञान की स्थिति को आगे बढ़ाता है और इस क्षेत्र में एसपीएच के भविष्य के विकास और अनुप्रयोगों के लिए आधार के रूप में काम करने की उम्मीद है।

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Abbreviations

ALE	A rbitrary L agrangian E ulerian
CEL	C oupled E ulerian L agrangian
CFD	C omputational F luid D ynamics
CFL	C ourant- F riedrichs- L ewy
CPT	C one P enetration T est
CRR	C yclic R esistance R atio
CSPM	C orrective S moothed P article M ethod
DDA	D iscontinuous D eformation A nalysis
DEM	D iscrete E lement M ethod or D igital E levation M odel
EFG	E lement F ree G alerkin
FDM	F inite D ifference M ethod
FEM	F inite E lement M ethod
FPM	F inite P article M ethod
FVM	F inite V olume M ethod
GIS	G eographic I nformation S ystem
GPU	G raphics P rocessing U nit
HPT	H ydraulic P rofilng T ool
HRSC	H igh R esolution S ite C haracterization
LBM	L attice B oltzmann M ethod
LCSD	L ean C oncentration S lurry D isposal
LEM	L imit E quilibrium M ethod
MCS	M onte C arlo S imulation
MLS	M oving L east S quare

MPI	M essage P assing I nterface
MPM	M aterial P oint M ethod
MPS	M oving P article S emi-Implicit
MSW	M unicipal S olid W aste
ODE	O rdinary D ifferential E quation
PDE	P artial D ifferential E quation
PEER	P acific E arthquake E ngineering R esearch
PFEM	P article F inite E lement M ethod
PGA	P eak G round A cceleration
RKPM	R eproducing K ernel P article M ethod
RPCM	R adial P oint C ollocation M ethod
RPIM	R adial P oint I nterpolation M ethod
RSPH	R andom S moothed P article H ydrodynamics
RVE	R epresentative V olume E lement
SBT	S oil B ehaviour T ype
SDG	S ustainable D evelopment G oal
SPH	S moothed P article H ydrodynamics
SPT	S tandard P enetration T est
SWCC	S oil W ater C haracteristic C urve
WVT	W eighted V oronoi T essellation

Symbols

$\mathbf{b}$	body force vector
c	solute concentration (mass of solute per unit volume of water)
c'	cohesion intercept
c'_0, c'_r	initial and residual values of cohesion intercept
c_s	speed of sound in the multiphase medium
C	solid concentration or normalised mass fraction
C_{CFL}	CFL coefficient
C_d	viscous damping coefficient
C_{ij}	stabilization term for particle pair i and j
$C_{ij,\pi}$	artificial viscosity stabilization term
$C_{ij,\omega}$	artificial stress stabilization term
C_k	Kozeny-Carman coefficient
C_s, C_p	speeds of shear and pressure waves in the soil
$\tilde{C}_{sr}$	storage coefficient
C_v	coefficient of consolidation
$d^\alpha()/dt$	material derivative on the reference frame of phase α
dS	elemental surface area
dV	elemental volume
D_r	relative density
$D_{r,cs}$	relative density at critical state
$\mathbf{D}$	coefficient of mechanical dispersion
$\mathbf{D}^*$	coefficient of molecular diffusion
D^*	scalar coefficient of molecular diffusion

$\mathbf{D}_h$	coefficient of hydrodynamic dispersion
$\mathbf{D}^e$	isotropic elasticity tensor
e	void ratio
e_0	void ratio at initial state
E	Young's modulus of the geomaterial
E^{mn}	$4\mathbf{x}_{ji}^m \mathbf{x}_{ji}^n / \mathbf{x}_{ji} ^2 - \delta^{mn}$
f	yield function
f_c	fines content
f_{c0}	initial fines content
$f_{c\infty}$	residual fines content after internal erosion is complete
f_{ce}	$(f_c - f_{c\infty}) / (f_{c0} - f_{c\infty})$
F	Mass of solute adsorbed per unit mass of solid phase
$\hat{F}_{ij}$	$\mathbf{x}_{ji} \cdot \nabla_i W_{ij} / \mathbf{x}_{ji} ^2$
$\tilde{F}_{ij}$	$\mathbf{x}_{ji} \cdot \tilde{\nabla}_i W_{ij} / \mathbf{x}_{ji} ^2$
g	vertical component of the acceleration due to gravity
g_0	plastic potential function
g_a, g_n, g_l	hydraulic constitutive model parameters
G	shear modulus of geomaterial
G_s	specific gravity
G_0	small strain shear modulus coefficient
h	smoothing length
h_p	pore water pressure head
h_{p0}	contraction rate parameter in PM4Sand model
H	total head or hardening/softening modulus
i, j	particle indices
I_1	first invariant of the effective stress tensor
$\mathbf{I}$	identity matrix
J_2	second invariant of the deviatoric stress tensor
$\mathbf{J}_f$	mass flux of water due to mechanical dispersion
$\mathbf{J}_\gamma$	mass flux of solute due to hydrodynamic dispersion

k_f	hydraulic conductivity coefficient (isotropic)
k_{f0}	reference hydraulic conductivity at zero solute concentration
$k_{f,sat}$	hydraulic conductivity coefficient (isotropic) in the saturated state
$k_{f,sat}^0$	initial saturated hydraulic conductivity coefficient (isotropic)
k_v, k_h	vertical and horizontal hydraulic conductivity
$\mathbf{k}_f$	hydraulic conductivity tensor
$\mathbf{k}_{f,sat}$	hydraulic conductivity tensor in the saturated state
$\mathbf{k}_{f,sat}^0$	initial saturated hydraulic conductivity tensor
K	bulk modulus of geomaterial
K_d	distribution coefficient
K_f	bulk modulus of water or fluid phase
K_0	coefficient of lateral earth pressure at rest
l_m	correlation length measured along the coordinate direction m
$\mathbf{L}_{ij}$	normalization matrix
m, n	coordinate directions
m_v	coefficient of volume compressibility
n	porosity
n_0	porosity at initial state
n_a, n_f, n_s	volume fraction of the air, water, and solid phases, respectively
n_{ss}, n_{se}, n_{sf}	volume fraction of the solid skeleton, erodible particles, and fluidised particles, respectively
$\hat{n}_e$	rate of volume fraction transfer due to internal erosion
$\mathbf{n}$	unit vector normal to the surface or boundary
$(N_1)_{60}$	corrected SPT blow count
p_a	pressure of the air phase
p_f	pressure of the water phase
$\bar{p}$	isotropic pressure = $I_1/3$
$\mathbf{q}_f$	Darcy velocity of the water phase
r_{ij}	distance between particles i and $j = \mathbf{x}_i - \mathbf{x}_j $
r_u	pore water pressure ratio

R	correlation coefficient between the properties at two points in the domain
R_d	retardation coefficient in solute transport model
R_i^{mn}, R_j^{mn}	artificial stress tensor for particles i and j
$\mathbf{R}^{fs}$	interaction force between the water and solid phases
s	deviatoric stress tensor
S_r	degree of saturation
S_{res}, S_{sat}	degree of saturation at a high suction head and full saturation, respectively
S_e	effective degree of saturation = $(S_r - S_{res})/(S_{sat} - S_{res})$
t	time
t_0	initial time
T_v	dimensionless time factor for one-dimensional consolidation
$\mathbf{v}_a, \mathbf{v}_f, \mathbf{v}_s$	velocity of the air, water, and solid phases, respectively
$\mathbf{v}_{ss}, \mathbf{v}_{se}, \mathbf{v}_{sf}$	velocity of the solid skeleton, erodible particles, and fluidised particles, respectively
$\mathbf{v}_{sji}$	velocity of particle j relative to particle i , $\mathbf{v}_{sji} = \mathbf{v}_{sj} - \mathbf{v}_{si}$
v_{su}, v_{pu}	particle speed of upward propagating shear and pressure seismic waves
v_{fxi}, v_{fyi}	velocity components of the free field region at particle i
vn, sn	velocity node, stress node
V	volume of the RVE
V_j	volume of particle j
V_a, V_f, V_s	volume of the air, water, and solid phases, respectively
V_{ss}, V_{se}, V_{sf}	volume of the solid skeleton, erodible particles, and fluidised particles, respectively
$W(\mathbf{x} - \mathbf{x}', h)$	smoothing kernel function
$W(\mathbf{x}_i - \mathbf{x}_j, h)$	smoothing kernel function value for particle pair i and j
W_{ij}	shorthand notation for $W(\mathbf{x}_i - \mathbf{x}_j, h)$
x, y	x and y coordinates of the position vector $\mathbf{x}$

$\mathbf{x}$	position vector
$\mathbf{x}_{ij}$	position of particle i relative to particle j , $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$
y	elevation head
z	depth from the surface
α_d	normalisation constant for the kernel function
α_l, α_t	longitudinal and transverse dispersivities
α_π, β_π	artificial viscosity coefficients
β_c, β_μ	density and viscosity difference factors
γ	inorganic solute in solute transport model
γ_w	unit weight of water
γ_{bulk}	bulk unit weight
γ_{sat}	saturated unit weight
Γ	boundary region
Γ_i	field variables of particle i other than position during leapfrog integration
$\delta(\mathbf{x} - \mathbf{x}')$	Dirac delta function
Δt	timestep for the temporal integration
$\Delta t_m, \Delta t_h$	timesteps based on the mechanical and hydraulic calculations
Δx	particle spacing
ϵ	strain tensor
ϵ_p	plastic strain tensor
$\bar{\epsilon}_p$	accumulated deviatoric plastic strain
ϵ_v	volumetric strain = $\text{tr}(\epsilon)$
$\epsilon_{p,v}$	volumetric plastic strain = $\text{tr}(\epsilon_p)$
η, ϵ	parameters controlling the magnitude of repulsive force due to artificial stress
η_s, η_p	dashpot coefficients in shear and normal directions
κh	radius of the kernel support domain
λ	plastic multiplier

λ_e	erosion constant
μ_f	viscosity of water
μ_{f0}, μ_s	viscosity of water corresponding to minimum and maximum solute concentration
ν	Poisson's ratio
$\xi, \eta, \bar{\eta}$	Drucker Prager model constants
ξ_l	coefficient of local damping
ξ_r	relative state parameter index
ρ_a, ρ_f, ρ_s	intrinsic density of the air, water, and solid phases, respectively
$\bar{\rho}_a, \bar{\rho}_f, \bar{\rho}_s$	partial density of the air, water, and solid phases, respectively
$\rho_{ss}, \rho_{se}, \rho_{sf}$	intrinsic density of the solid skeleton, erodible particles, and fluidised particles, respectively
$\bar{\rho}_{ss}, \bar{\rho}_{se}, \bar{\rho}_{sf}$	partial density of the solid skeleton, erodible particles, and fluidised particles, respectively
ρ_m	density of the multiphase mixture
ρ_w	density of water
ρ_{f0}, ρ_s	density of water corresponding to minimum and maximum solute concentration
$\hat{\rho}_e$	mass transfer due to internal erosion
σ	total stress tensor
σ'	effective stress tensor
$\sigma_a, \sigma_f, \sigma_s$	intrinsic stress tensor of the air, water, and solid phases, respectively
$\bar{\sigma}_a, \bar{\sigma}_f, \bar{\sigma}_s$	partial stress tensor of the air, water, and solid phases, respectively
σ_v	vertical overburden stress
σ'_{v0}	vertical effective stress at initial state
σ'_{tr}	trial stress state assuming fully elastic material response
σ'_n	effective stress state at timestep n
τ_{xy}	shear stress
ϕ'	angle of shearing resistance

ϕ'_0, ϕ'_r	initial and residual values of angle of shearing resistance
ψ	dilation angle
ω	spin tensor or solute mass fraction (mass of solute per unit mass of water)
ω_{f0}, ω_s	mass fraction corresponding to minimum and maximum solute concentration
Ω	domain region
$\nabla_i W_{ij}$	kernel gradient with reference to particle i
$\tilde{\nabla}_i W_{ij}$	corrected kernel gradient with reference to particle i