

A THESIS ON
"ON CERTAIN HYPERGEOMETRIC FUNCTIONS AND POLYNOMIALS
OF ONE AND MORE VARIABLES"

By
Hari Ram Sharma
Department of Mathematics
Indian Institute of Technology
New Delhi

Submitted to the Indian Institute of
Technology, New Delhi for the award
of the Degree of Doctor of Philosophy
in Mathematics
1969

4

C E R T I F I C A T E

This is to certify that the thesis entitled "On Certain Hypergeometric Functions and Polynomials of one and more Variables" which is being submitted by Mr. Hari Ram Sharma for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

M.K. Jain
(M.K. Jain)
Head: Department of Mathematics
Indian Institute of Technology
Delhi

A C K N O W L E D G E M E N T S

I wish to express a profound sense of gratitude to Professor M.K. Jain, M.A., D. Phil., D.Sc. now visiting Professor to the University of Illinois, Urbana (U.S.A.) for his kind supervision, generous encouragement and inspiration to me. I am extremely thankful and deeply indebted to Shri H.L. Marocha, Assistant Professor, Department of Mathematics, Indian Institute of Technology, New Delhi for his very valuable guidance and constant help throughout the preparation of this thesis. Without his personal interest in my work it would not have been possible to complete the work.

I am thankful to Professor M.P. Singh, Ph.D., Head of the Department of Mathematics, Indian Institute of Technology, New Delhi for his keen interest in my work.

My thanks are also due to the authorities of Indian Institute of Technology, New Delhi and Council of Scientific and Industrial Research, New Delhi for providing me with all research facilities and giving me the financial help during the course of these investigations.

Nonetheless, I am very much thankful to Mr. P.C. Sharma and Mr. J.L. Gupta for giving me their valuable time in reading the manuscript, making detailed corrections and many helpful suggestions.

I will be failing in my duty if I don't record my appreciation of the inspiration given to me by Mrs. Shashi Sharma.

I thank Mr. D.R. Joshi for his commendable work in typing the manuscript.

Department of Mathematics,
Indian Institute of Technology,
Haus Khas, New Delhi-29

H. Sharma
30.8.69
(Hari Ram Sharma)

C O N T E N T S

Chapter		Page
I	INTEGRAL EXPRESSIONS AND GENERATING FUNCTIONS BY MEANS OF FRACTIONAL DERIVATIVES	1 - 40
	1. Introduction	3
	2. Rules for fractional integration and differentiation	7
	3. Theorem on term by term fractional differentiation (integration)	8
	4. Derivations of (1.13), (1.14), (1.15)	10
	5. Transformation of (1.13), (1.14) and (1.15) by fractional integration by parts	15
	6. Generating functions by fractional derivatives	25
	7. Generating relations involving Jacobi polynomials	30
	8. Generating relations involving Laguerre polynomials	34
	9. References	39
II	SUMMATION FORMULAE FOR TRIPLE SERIES	41 - 56
	1. Introduction	42
	2. Theorem analogous to (1.1)	44
	3. Theorems analogous to (1.2) and (1.3)	47
	4. A transformation formula of hypergeometric function of three variables	50
	References	56
III	ON JACOBI POLYNOMIALS	57 - 82
	1. Introduction	58
	2. Integral representation for the product of four Jacobi polynomials	60

	3. Generating functions involving Jacobi polynomials	64
	4. Generating functions involving Jacobi functions	67
	5. Generating functions involving Jacobi polynomials and Jacobi functions	74
	References	82
IV	ON GENERALIZED HYPERGEOMETRIC POLYNOMIALS	83 - 102
	1. Introduction	84
	2. A generating function	85
	3. Some more generating functions	87
	4. An integral involving Rice polynomials	91
	5. Recurrence relations	96
	6. Some characterisations of the Generalized hypergeometric polynomials	98
	References	102
V	ON BESSEL POLYNOMIALS	103 - 122
	1. Introduction	104
	2. Generating functions	105
	3. A characterization of the Bessel polynomials	114
	References	121
VI	ON A THEOREM BY BROWN AND CHRISTOFFEL-DARBOUX FORMULA	123 - 148
	1. Introduction	125
	2. Generalization of the theorem by Brown	126
	3. On Christoffel-Darboux formula	131
	4. Summation formula for the Laurecella's function F_A	136
	5. Some Results by means of (4.2)	138
	6. Generating functions involving Appell function F_2	142
	References	148