SOME PROBLEMS ON
STATISTICAL THEORY OF TURBULENCE

by
Rangadhar Dash
Department of Mathematics,
Indian Institute of Technology,
New Delhi

Submitted to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics.

1969

CERTIFICATE

This is to certify that the thesis entitled 'Some Problems on Statistical Theory of Turbulence' which is being submitted by Mr. Rangadhar Dash for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last two years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Prem Kumar
13.10.68

(Prem Kumar)
Department of Mathematics,
Indian Institute of Technology,
Hauz Khas, New Delhi-29.

ACKNOWLEDGEMENTS

I am profoundly grateful to Dr. Prem Kumar M.A., Ph.D., Department of Mathematics, Indian Institute of Technology, Delhi for his valuable guidance throughout my research work. But for his keen interest, generous encouragement and inspiration to me, this work would not have been possible.

I express my deep sense of gratitude to Dr. M.P. Singh M.Sc., Ph.D., Head of the Department of Mathematics and Dr. M.K. Jain, D.Phill., D.Sc., visiting Professor at the University of Illinois, U.S.A. for their interest in my progress.

I am also highly thankful to Prof. R.G. Deissler, Chief of the Fundamental Heat Transfer Branch, NASA, Ohio, U.S.A., for his suggestions and encouraging comments in the preparation of my work.

I would also like to record my deep sense of gratitude to my father and brothers for their patience and affection which have provided impetus to my work. My thanks are also due to my wife for her help in the proof reading and correction of my papers.

I am deeply obliged to the authorities of the Indian Institute of Technology, Delhi for their kindness in granting me a research fellowship and providing me facilities during my research here.

I also thank Mr. Ranjit Kumar for his excellent typing of the thesis.

R Dash

(R. Dash),
Department of Mathematics,
Indian Institute of Technology,
New Delhi.

Synopsis

This thesis incorporating some aspects of the statistical features of turbulence includes six chapters. The introductory first chapter deals, in brief, with the concept of turbulence, the necessity of statistical approach for discussing the turbulent phenomena and the justification for its use, the present-day scope of turbulence, the gradual development of some related ideas and outstanding contributions of some of the pioneers in the field.

The second chapter presents a generalized study of the multipoint, multitime velocity correlations with reference to a decaying homogeneous turbulence, and the discussion has been made in the light of Deissler's usual treatment of correlations. Equations involving correlations between the fluctuating quantities at four points and at four times are constructed from the momentum and the continuity equations. To make the set of equations determinate, the terms containing the quintuple correlations are neglected in comparison with those containing the quadruple ones. In the process, calculations have been made for the energy-spectrumfunction which represents the contributions from various wane-numbers to the total energy, and the energy transfer function which is responsible for the transfer of energy between wave-numbers. The solutions so obtained give the state of the homogeneous turbulence between its initial and final periods, and reduce to the multipoint, onetime results of Deissler, when all the time intervals are simultaneously zero.

The third chapter deals with the problem of temperature fluctuations in axisymmetric turbulence. Here the discussion is based on Chandrasekhar's theory of axisymmetric turbulence. The resulting expression, which has been developed from the energy equation for an incompressible fluid with constant properties and negligible frictional heating, for the fluctuating temperature correlations shows an exponential decay and brings to light three distinct types of axisymmetric turbulence - the cylindrical axisymmetric turbulence, the conical axisymmetric turbulence and the circular axisymmetric turbulence. Moreover, the cylindrical axisymmetric turbulence which is usually found in nature is also predicted by the present study, and the same prediction has also been made previously by Batchelor and

Chandrasekhar.

In the fourth chapter dealing with the nonisotropic turbulent scalar field, a dynamic equation for the double concentration correlation has been developed from the mass transfer equation with constant molecular transport co-efficient. With the help of Heisenberg's assumption, and under different turbulent situations corresponding energy spectra have been found and it is found that they exhibit the same power-laws as shown by the velocity energy-spectra.

In the fifth chapter, investigation has been made for the concentration fluctuations in a steady state shear flow turbulence in which the weak turbulence approximation has been used to find expressions for the correlation terms. Generalized two point correlation equations are constructed from the mass transfer equation by the usual method used by Von Karman and Howarth. Since the approximation limits the analysis to low Reynolds number, results are obtained from them by expanding them in power series in each of the space veriables to have algebraic expressions for the correlations. The result obtained for the concentration-correlation may be used to estimate its variations with different values of Schmidt numbers.

The sixth chapter which is also the concluding one of the thesis, presents a discussion of the statistical behaviour of a first-order chemical reaction in a decaying homogeneous turbulence before the final period. Results are obtained, after using the usual technique of Deissler, for the convective transfer function, the spectral 'energy' function, total concentration fluctuation 'energy', and the corresponding microscales. Since, as a first approximation theory, the theory presented here is restricted to large convection/diffusive ratios and since (for a given Reynolds number) increasing Schmidt number signifies an increasing value of this ratio, it might be expected that the theory would apply well for higher values of Schmidt number.

List of Papers

1. Multipoint, multitime velocity correlations and decay of Homogeneous Turbulence

   Accepted for publication in 'The Physics of Fluids'. This paper comprises the second chapter of the thesis.

2. Temperature fluctuations in axisymmetric turbulence

   Accepted for publication in the 'Tensor'. This paper constitutes the third chapter.

3. Non-isotropic turbulent scalar field.

   It comprises the fourth chapter of the thesis.

4. Concentration fluctuations in steady-state shear-flow turbulence.

   Communicated to the 'International Journal of Heat and Mass transfer'.

   It is the firth chapter of the thesis.

5. Statistical behaviour of a first-order chemical reaction in a decaying homogeneous turbulence before the final period.

   To appear in the Int. J. of Heat & Mass Transfer.

   It comprises the sixth chapter of the thesis.

C O N T E N T S