

**STABILITY OF COMPLIANT PIPE FLOW  
WITH AXISYMMETRIC AND NON-  
AXISYMMETRIC DISTURBANCES**

**By**

**MUNENDRA KUMAR**

**Submitted**

**In fulfilment of the requirements of the degree of**

**Doctor of Philosophy**

**to the**



**DEPARTMENT OF APPLIED MECHANICS  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

**JUNE 2004**

pp fluid dynamics

LIBRARY  
No. TH-3191



TH  
621.64:536.2  
MUN-S

# CERTIFICATE

This is to certify that the thesis entitled “Stability of compliant pipe flow with axisymmetric and non-axisymmetric disturbances” being submitted by Mr. Munendra Kumar for the award of the degree of Doctor of Philosophy, to the Indian Institute of Technology, Delhi is a record of bonafide work carried out by him under our guidance and supervision. The material embodied in this thesis, unless acknowledged otherwise, has not been submitted in part or in full for any other Diploma or Degree of any University.



(P. K. Sen)  
Professor

Department of Applied Mechanics  
Indian Institute of Technology, Delhi  
New Delhi – 110016, India.



(A. K Raghava)  
Associate Professor

Department of Applied Mechanics  
Indian Institute of Technology, Delhi  
New Delhi – 110016, India.

## **ACKNOWLEDGEMENTS**

There are a number of persons to whom I am indebted in the accomplishment of my thesis. First of all, I am extremely grateful to my supervisor, Professor P. K. Sen without whose valuable suggestions and encouraging guidance the research work would not have been completed. Professor Sen served as a beacon, whose illuminating guidance made the research work proceed in a smooth and efficient manner. I would like to express my gratitude to Dr. A. K. Raghava for his guidance. Without his support, the completion of the thesis would have been difficult. I express my sincere thanks to Professor (Mrs.) Padma Vasudevan Sen for extending me the much-needed co-operation in terms of encouragement.

I also express my thanks to Professor P. W. Carpenter of University of Warwick, U. K., for valuable discussions and comments, during his visits to India, and, during the course of my work.

I am thankful to my friend, Dr. S. Hegde for providing me moral support and, for hardware and software support in computing, during the phase of completion of the work. I am also thankful to Mr. P S Josan and Mr. S. K. Vyas for their moral support and encouragement.

I also owe my gratitude to my wife Dr. Meenu Singh and children Sangam and Shilpi, who have given me their utmost co-operation.

Lastly, I am grateful to all those persons who have helped me directly or indirectly during the course of my work.

*M. Kumar.*  
(Munendra Kumar)

## ABSTRACT

The present work discusses the hydrodynamic stability of compliant pipe flow, considering a visco-elastic pipe, with an outer rigid shroud, and, with Hagen Poiseuille flow through the pipe. The work includes the normal compliance studies by the Sen and Arora method, physical realisability studies by the equivalent plate-spring model, normal compliance studies for the full visco-elastic wall, and combined normal plus tangential compliance studies. Both the axisymmetric and non-axisymmetric disturbances have been considered. The coupled fluid-solid equations have been solved numerically by extensions of the methods of Sen and Venkateswarlu (1983), Sen, Venkateswarlu and Maji (1985) and Sen and Arora (1988). Energy methods have also been extensively used.

Fluid flow in a pipe with flexible walls generally occurs in nature, e.g., in biological systems like flow of blood and other fluids in the body. Such a system also occurs in industrial applications for internal flow through hollow fibres, reactors and membranes. It is found that such a system is unstable to both axisymmetric and non-axisymmetric disturbances, although, the rigid pipe problem is stable to all infinitesimal disturbances. For a given azimuthal wavenumber, it is found that there are broadly two unstable mode classes, for both the axisymmetric and non-axisymmetric instabilities. One is a solid based flow induced surface instability, while the other is a fluid based surface instability that asymptotes to the least damped rigid wall mode as the thickness of the visco-elastic wall tends to zero. All modes are stabilized, to different degrees, by the viscoelastic wall viscosity.

The Sen and Arora (1988) method is used for the only-normal compliance problem. Here the normalized disturbance velocity at the wall,  $\bar{\phi}_w$ , is used as a kinematic wall parameter. Physical realisability of modes can be found out by back calculating the values of the surface wave speed  $c_0^2$  and damping  $d$  for an equivalent plate spring model: realisability given by the criterion  $c_0^2 > 0$  and  $d > 0$ . All rigid type (deviant of rigid wall) modes are centerline modes, and are *stable*. Unstable modes are basically either static divergence modes or transitional modes.

In the “force method”, we study the neutral stability curves for different values of the Kumaran parameter  $\Gamma$ , and the solid to fluid viscosity ratio  $\mu_r$ , for axisymmetric and non-axisymmetric disturbances. However, the normal compliance modes are more unstable and are substantially different from the normal plus tangential compliance modes. It is also observed that wall dissipation tends to damp all the modes. Static divergence (SD) modes are also found in the  $n = 0, 1$  modes, where  $n$  is the azimuthal wavenumber.

From the “energy methods” we obtain the significance of each term in the energy balance. In the rigid wall problem, the production term  $I_2$  and the dissipation term  $I_3$  are *both negative*; thus the rigid wall modes are always stable. Our results based on the energy method provides a much deeper insight into the relevant mechanisms involved. We are able to see for which class of modes  $I_2$  is the dominant production term, and for which class of modes the tractive work term  $B_4$  is the dominant production term. We are also able to see which are the main dissipation terms or redistribution terms for the fluid-side and the solid-side for various classes of modes.

# Contents

i

## List of Symbols

vi

## Chapter 1

### INTRODUCTION

1.1	Hydrodynamic stability theory	1
1.2	Rigid pipe flow	2
1.3	Compliant surface	3
1.4	Compliant pipe flow	4
1.4.1	Collapsible tubes	4
1.4.2	Non-collapsible tubes	5
1.5	The present problem	6
1.5.1	The only-normal compliance problem	7
1.5.2	The full normal plus tangential compliance problem	7
1.5.3	Energy method	7

## Chapter 2

### REVIEW OF EARLIER WORK AND RELATED LITERATURE

2.1	Rigid pipe flow	9
2.2	Flow over flat compliant surfaces	13
2.3	Compliant pipe flow	16
2.3.1	Review of the works of Kumaran, and coworkers	16
2.3.2	Review of the works of Hamadiche and Gad-el-Hak	20
2.3.3	Review of the works of Gibson, and, Gajjar, Gibson and Sen	21
2.4	Overview of salient past results of compliant pipe flow	22
2.5	Motivation of the present work	23

## **Chapter 3**

### **FORMULATION OF THE PROBLEM**

3.1	Introduction	24
3.2	The disturbance equations for the fluid	25
3.3	The disturbance equations for the solid	27
3.4	Boundary conditions for the combined fluid-solid problem	29
3.5	Procedure of solution by the Force method	32
3.5.1	Only normal compliance problem	33
3.5.1.1	The kinematic or Sen and Arora model	33
3.5.1.2	Physical realisability based on Sen and Arora model	33
3.5.1.3	Viscoelastic wall supported by a thin plate at the interface	35
3.5.2	The full Normal plus Tangential compliance model	35
3.6	Energy method	35
3.6.1	Energy equations for the fluid-side	36
3.6.2	Energy equations for the solid-side	39
3.7	Model of normal, and normal plus tangential compliance problems	42

## **Chapter 4**

### **SOLUTION METHODS AND NUMERICAL APPROACH**

4.1	Introduction	44
4.2	Solution methods	44
4.3	Solution procedure	47
4.3.1	Rigid wall solution	47
4.3.2	Solution based on the Sen and Arora method	48
4.3.3	Solution of the combined fluid-solid N problem	49
4.3.4	Solution of the normal plus tangential compliance problem	50

# Chapter 5

## RESULTS AND DISCUSSIONS

5.1	Introduction	52
5.A	Sen and Arora method	54
5.A.1	Axisymmetric disturbances	54
5.A.2	Non-axisymmetric disturbances	58
5.2	General conclusions based on the Sen and Arora method	59
5.B	The force method	73
5.B.1	Effect of thickness $H$ of the viscoelastic material	75
5.B.2	The HGB N+T modes, and N modes, $n=0$	78
5.B.2.1	Effect of $\Gamma$ on HGB N+T, and N, $n = 0$	78
5.B.2.2	Effect of e.t. on HGB N+T, $n = 0$	79
5.B.2.3	Effect of $\mu_r$ on HGB N+T, and N, $n=0$	79
5.B.2.4	Sample eigenfunctions for HGB N+T, $n = 0$	80
5.B.3	The HGA N+T modes, and N modes, $n=0$	80
5.B.3.1	Effect of $\Gamma$ on HGA N+T, and N, $n = 0$	80
5.B.3.2	Effect of e.t. on HGA N+T, $n = 0$	81
5.B.3.3	Effect of $\mu_r$ on HGA N+T, and N, $n=0$	81
5.B.3.4	Sample eigenfunctions for HGA N+T, $n = 0$	81
5.B.4	The HGB N+T modes, and N modes, $n=1$	82
5.B.4.1	Effect of $\Gamma$ on HGB N+T, and N, $n = 1$	82
5.B.4.2	Effect of e.t. on HGB N+T, $n=1$	82
5.B.4.3	Effect of $\mu_r$ on HGB N+T, and N, $n=0$	82
5.B.4.1	Sample eigenfunction for HGB N+T, $n = 0$	83
5.B.5	The HGA N+T modes, and N modes, $n=1$	83
5.B.5.1	Effect of $\Gamma$ on HGA N+T, and N, $n = 1$	83
5.B.5.2	Effect of e.t. on HGA N+T, $n = 1$	84
5.B.5.3	Effect of $\mu_r$ on HGA N+T, and N, $n=1$	84
5.B.5.4	Sample eigenfunction for HGA N+T, $n = 0$	84

5.B.6	The N+T modes, and N modes, $n=2$	84
5.B.6.1	Effect of $\Gamma$ on N+T, and N, $n = 2$	84
5.B.6.2	Effect of e.t. on N+T, $n = 2$	85
5.B.6.3	Effect of $\mu_r$ on N+T, and N, $n=2$	85
5.B.7	The N+T modes, and N modes, $n=3$	85
5.B.7.1	Effect of $\Gamma$ on N+T, and N, $n = 3$	85
5.B.7.2	Effect of e.t. on N+T, $n = 3$	86
5.B.7.3	Effect of $\mu_r$ on N+T, and N, $n=3$	86
5.B.8	The N+T modes, and N modes, $n=4$	86
5.B.8.1	Effect of $\Gamma$ on N+T, and N, $n = 4$	86
5.B.8.2	Effect of e.t. on N+T, $n = 4$	87
5.B.8.3	Effect of $\mu_r$ on N+T, and N, $n=4$	87
5.B.9	Summary of conclusions based on the force method	88
5.C	The energy method	122
5.C.1	Rigid wall modes, $n = 0, 1$	123
5.C.2	Axisymmetric disturbances, $n = 0$	123
5.C.2.1	The HGA N+T mode, $n=0$	124
5.C.2.2	The HGB N+T mode, $n=0$	126
5.C.2.3	The N mode, $n=0$	128
5.C.3	Non-axisymmetric disturbances, $n = 1$	129
5.C.3.1	The HGA N+T mode, $n=1$	129
5.C.3.2	The HGB N+T mode, $n=1$	131
5.C.3.3	The N mode, $n=1$	132
5.C.4	Non-axisymmetric disturbances, $n = 2$	134
5.C.4.1	The N+T mode, $n=2$	134
5.C.4.2	The N mode, $n=2$	135
5.C.5	Non-axisymmetric disturbances, $n = 3$	137
5.C.5.1	The N+T mode, $n=3$	137
5.C.5.2	The N mode, $n=3$	138
5.C.6	Non-axisymmetric disturbances, $n = 4$	139
5.C.6.1	The N+T mode, $n=4$	139
5.C.6.2	The N mode, $n=4$	141

5.C.7 The HGB N+T SD modes, $n = 0$	142
5.C.8 The HGB N+T SD modes, $n = 1$	143
5.C.9 General conclusions based on the energy method	145

## Chapter 6

### CONCLUSIONS

6.1 Summary of results	180
6.1.1 The Sen and Arora method	180
6.1.2 The force method	181
6.1.3 The energy method	183
6.2 Overall conclusions	187
6.3 Scope for future work	188

### APPENDIX

A.1 Introduction	190
A.2 Axisymmetric case	191
A.2.1 Centerline boundary conditions	192
A.2.2 Boundary conditions at the rigid shroud	193
A.2.3 Interface boundary conditions for the fluid-side	193
A.2.4 Interface boundary conditions for the solid-side	195
A.3 Non-axisymmetric case	198
A.3.1 Centerline boundary conditions	198
A.3.2 Boundary conditions at the rigid shroud	200
A.3.3 Interface boundary conditions for the fluid-side	200
A.3.4 Interface boundary conditions for the solid-side	204
A.3.5 Rigid wall problem	208

REFERENCES	210
------------	-----