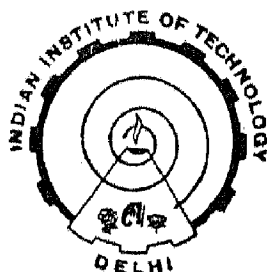


GENERALIZED PRIMITIVE RINGS AND CORRESPONDING GROUP RINGS

By
ANAND BALA

THESIS SUBMITTED TO THE
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
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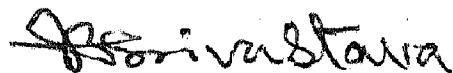


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1983

CERTIFICATE

This is to certify that the thesis entitled 'Generalized Primitive Rings and Corresponding Group Rings' being submitted by Miss Anand Bala to the Indian Institute of Technology, Delhi for the award of the Ph.D. degree in Mathematics, is a record of bonafide research work carried out by her under my supervision for the last four and a half year and to the best of my knowledge it has reached the standard, fulfilling the requirements of the regulations relating to the degree.

I further certify that the results contained in this thesis have not been submitted, either in part or in full, to any other University or Institute for the award of any Diploma or Degree.



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ABSTRACT

Throughout the thesis ring will mean associative ring with identity $1 \neq 0$. A ring R is left primitive if it possesses a faithful irreducible left R -module. Every primitive ring is prime and a prime ring with non-zero socle is primitive. The Jacobson-Chevalley density theorem gives that a ring R is primitive if and only if R is isomorphic to a dense ring of linear transformations of a vector space over a division ring.

During the last fifteen years, there has been a significant development related to prime and primitive rings and a serious study of generalized primitive rings has emerged. A recent development is the study of weakly primitive, endo-primitive, K -primitive and H -primitive rings and corresponding radicals. A basic tool has been the fundamental work of S.A. Amitsur on Morita contexts.

We shall denote by $K[G]$ the group algebra of the group G over the field K . A group algebra $K[G]$ is prime if and only if G does not contain any non-identity finite normal subgroup and this is the case if and only if the FC-subgroup $\Delta(G)$ is torsion-free Abelian. Lot of work has been done on the Jacobson radical, prime radical and N^* -radical of group algebras. A serious study of primitive group rings has been going on and lot of work has been done in this area.

There seems to have been virtually no progress in the study of generalized primitive group rings but for sporadic results here and there. The aim in this thesis is to study generalized primitive rings and group rings and also related topics.

In Chapter I, we have given a coherent account of the basic results on group rings, generalized primitive rings and Morita contexts which are needed for our further work. Here some of the results are new and others are either known or can be obtained from known results after some work.

In Chapter II, we have studied the problem of determining the fields K and groups G for which $K[G]/J(K[G])$ is a primitive algebra. Here $J(K[G])$ denotes the Jacobson radical. For convenience sake, we call a ring R $*$ -primitive if $R/J(R)$ is a primitive ring. In this chapter we have studied $*$ -primitive rings and group rings. A large varieties of groups G have been obtained for which the group algebra $K[G]$ is $*$ -primitive. If $K[G]$ satisfies a polynomial identity, then a complete characterization of $*$ -primitivity has been worked out. Many other results are given.

In Chapter III, rings, having faithful module of a given type, are studied. We have investigated properties of characteristically primitive rings. The class of all characteristically primitive rings has been shown to be a normal

class of prime rings. Also, H -primitive rings are shown to constitute a normal class. The corresponding radicals are normal radicals. Our results use the basic tool of Morita contexts. Chapter IV deals with the generalized primitive group rings. Mainly K -primitive group algebras are studied. Some other prime group rings and examples of interest are also given.

Finally, in the last Chapter V, we have studied finite normalizing extensions, strongly prime group rings and connection amongst various radicals. Mainly we have studied strongly prime group rings. Here the main tool is the knowledge of various intersection theorems for group rings. We have worked on the conjecture that $K[G]$ is strongly prime if and only if G does not contain any non-identity locally finite normal subgroup. Let H be a normal subgroup of G and let K be a field such that $K[H]$ is strongly prime and $I \cap K[H] \neq (0)$ for every non-zero ideal I of $K[G]$, then it has been shown that $K[G]$ is also strongly prime. The cases of nilpotent, solvable and FC-groups are completely settled. Also suitable Wreath products are settled. A large class of groups is constructed whose group algebras are strongly prime.

CONTENTS

	<u>Page</u>
ACKNOWLEDGEMENTS	i
LIST OF SYMBOLS	ii
ABSTRACT	v
CHAPTER 0 INTRODUCTION	1
CHAPTER 1 GROUP RINGS, PRIME RINGS AND MORITA CONTEXTS	6
1. GROUP RINGS	7
2. PRIME RINGS	13
3. MORITA CONTEXTS	25
CHAPTER 2 GROUP ALGEBRAS PRIMITIVE MODULO JACOBSON RADICAL	36
1. *-PRIMITIVE RINGS	36
2. *-PRIMITIVE GROUP ALGEBRAS	42
CHAPTER 3 GENERALIZED PRIMITIVE RINGS	59
1. CHARACTERISTICALLY PRIMITIVE RINGS	60
2. H-PRIMITIVE RINGS	67
CHAPTER 4 GENERALIZED PRIMITIVE GROUP RINGS	71
1. K-PRIMITIVE GROUP RINGS	72
2. ENDOPRIMITIVE GROUP RINGS	81

CHAPTER V	PRIME GROUP ALGEBRAS	84
	1. NORMALIZING EXTENSIONS	85
	2. STRONGLY PRIME GROUP RINGS	90
	3. RADICALS	102
REFERENCES		108
BIO-DATA		116
