STATIC AND DYNAMIC ANALYSES OF LAMINATED AND FUNCTIONALLY GRADED SANDWICH PANELS USING A LAYERWISE THEORY

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by

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Certificate

This is to certify that the thesis entitled "**Static and Dynamic Analyses of Laminated and Functionally Graded Sandwich Panels Using a Layerwise Theory**" being submitted by **Shashank Pandey** to the Indian Institute of Technology Delhi, New Delhi for the award of the degree of **Doctor of Philosophy**, is a record of original, bonafied research work carried out by him under my supervision and guidance. The thesis work in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

Date :

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Abstract

Laminated composite and sandwich plate and shell panels are widely used in civil, mechanical, automobile and marine engineering applications due to their high stiffness and strength to weight ratios. A typical sandwich structure consists of one or more high strength and high stiffness facesheets bonded to one or more cores. Such sandwich structures are weak in shear due to the presence of soft cores which have low shear modulus compared to their extensional rigidity. The effect of shear deformation is quite significant in case of sandwich structures as material properties vary drastically between the core and facesheets and the accuracy of the results for sandwich structures largely depends on the computational model adopted. Due to the sudden change in material properties, the stresses across the interface are found to be having discontinuity. To alleviate such problems, a new class of materials known as functionally graded materials (FGM), was proposed. The FGMs are made of two different materials, viz. metal and ceramic, and the elastic properties are gradually varied across the thickness so that the stress distribution is continuous across the interface. Functionally graded materials are often used in high temperature applications such as nuclear, space and many other applications.

This work presents finite element formulation based on layerwise displacement fields to analyze static and dynamic behavior of laminated and FGM sandwich plate and shell panels. A new layerwise theory is developed in which the higher-order in-plane displacement fields are expanded using Taylor's series for the core and first-order displacement field is assumed for both upper and lower facesheets maintaining the displacement continuity at the layer interfaces. The transverse displacement is considered to be constant along the thickness of the panel. Eight-noded isoparametric elements with 13 and 9 nodal unknowns per node for higher-order and first-order layerwise formulations, respectively are employed for the layerwise formulation. The presented finite element formulation requires a C^0 continuity, which is simple to obtain. Hamilton's principle is used in order to formulate governing static and free vibration equations. In the present study, the elastic properties of FGM under thermal condition are considered to be temperature dependent and the temperature distribution along the thickness of the panel is considered as nonlinear. The effective material properties of FGM are obtained using two micro mechanical models, rule of mixture (ROM) and the Mori-Tanaka (MT)

homogenization scheme. The material composition of the FGM are varied by changing the volume fraction index of the constituent materials.

Since FGM are employed in high temperature applications, study of thermally induced vibrations in such panels due to thermal shock is essential for its application in high temperature. Therefore, in this work, behavior of FGM sandwich plate and shell panels subjected to different thermal shock conditions is also investigated. The solution for temperature profile is obtained using central difference method and Crank-Nicolson time integration procedure. Further, the dynamic equations are solved by using Newmark average acceleration time marching scheme to obtain transient displacement and stresses.

The results obtained from the present layerwise formulations are first compared with benchmark solutions available in the literature in order to establish the correctness of the formulation. Convergence studies are carried out to determine appropriate mesh sizes required for static and dynamic analyses of laminated and functionally graded sandwich panels. After establishing the accuracy of the formulations, parametric study is conducted to investigate effects of geometric parameters, elastic properties, thermal gradients, sandwich configuration and boundary conditions on the static and free vibration behavior of sandwich plate and shell panels, as required appropriately. It is shown that the proposed layerwise formulations are simple, straightforward and can accurately predict static and dynamic behavior of both laminated and functionally graded sandwich panels for a wide range of problems.

सारांश

लैमिनेटेड और सैंडविच प्लेट और शैल पैनलों के उच्च कठोरता और ताकत के अनुपात में वजन कम होने के कारण सिविल, मैकेनिकल, ऑटोमोबाइल और समुद्री इंजीनियरिंग में प्रयोग में लाये जाते है। एक प्ररूपी सैंडविच की संरचना दो सख्त पदार्थ से बनी ऊपरी तथा निचली सतह के मध्य एक मुलायम पदार्थ से होती है। ऊपरी एवम निचली सख्त सतह तथा मुलायम मध्य को फैसशीट एवम कोर क्रमशः कहा जाता है। नरम कोर के कारण सैंडविच पैनलों का यंग मापक अपरूपण गुणांक से अधिक होता है फलस्वरूप इस तरह के पैनलों अपरूपण सामर्थ्य अत्य अधिक कम होता हैं। इस तरह के सैंडविच पैनलों में अपरूपण विरूपण अभिप्रायपूर्ण होने के कारण सैंडविच पैनलों के विश्लेषण में सटीक गणितीय गणना महत्वपूर्ण योगदान हैं। फैसशीट एवम कोर के भिन्न पदार्थ गुण होने के कारण दोनों सतह पर प्रतिबल अत्यधिक भिन्न होता हैं। कार्यात्मक वर्गीकृत पदार्थ जिसे फंक्शनलली ग्रेडेड मटेरियल के नाम से जाना जाता हैं वो आधुनिक पदार्थ हैं जिसका निर्माण धातु एवं चीनी मिट्टी से होता हैं एवं पदार्थ गुण का बदलाव ऊपर से निचे सहज तरीके से होता हैं फलस्वरूप प्रतिबल का बदलाव भी उसी दिशा में सहजता से होता है। फंक्शनलली ग्रेडेड मटेरियल का अनुप्रयोगों परमाणु, अंतरिक्ष और कई अन्य उच्च तापमान अनुप्रयोगों में किया जाता हैं।

इस शोधलेख में परिमित तत्व विधि एवं परत दर परत विस्थापन क्षेत्र का प्रयोग करते हुए लैमिनेटेड और सैंडविच प्लेट और शैल पैनलों का स्थिर और गतिशील व्यवहार का विश्लेषण किया गया हैं। एक नया परत दर परत सिद्धांत विकसित किया गया हैं, जिसमें कोर के लिये उच्च क्रम का विस्थापन क्षेत्र तथा फैसशीट के लिये प्रथम क्रम का विस्थापन क्षेत्र की कल्पना की गयी हैं तथा इसका संबोधन हायर आर्डर लयेरवाइस थ्योरी से किया गया हैं । परिमित तत्व विधि जिसमें कोर तथा फैसशीट के लिये प्रथम क्रम का विस्थापन क्षेत्र की कल्पना की गयी हैं संबोधन फर्स्ट आर्डर लयेरवाइस थ्योरी से किया गया हैं, अनुप्रस्थ विस्थापन पैनल की मोटाई के साथ निरंतर माना गया हैं । आठ नोडेड आइसोपेरामेट्रिक एलिमेंट जिसमें तेरह तथा नौ डिग्री ऑफ़ फ्रीडम एवम हायर आर्डर लयेरवाइस थ्योरी तथा फर्स्ट आर्डर लयेरवाइस थ्योरी का क्रमशः प्रयोग करके लयेरवाइस प्रतिरूप का निर्माण किया गया हैं । वर्तमान परिमित तत्व विधि को मात्र C^0 निरंतरता की आवशकता है , जिसको सहजता से प्राप्त किया जा सकता है । हैमिलटन सिद्धांत का प्रयोग करके मुख्य अवकल समीकरण प्राप्त करके स्थिर और गतिशील विश्लेषण किया गया है । इस शोध कार्य में फंक्शनलली ग्रेडेड मटेरियल में प्रयोग होने वाले पदार्थो के गुणों की ग़णाना रूल ऑफ़ मिक्सचर एवम मोरी-तनाका पद्धति द्वारा की गयी है तथा पदार्थो के गुण, तापमान पर भी निर्भर हैं । तापमान का परिवर्तन की मोटाई की दिशा में गैररेखा तरीके से माना गया है। फंक्शनलली ग्रेडेड मटेरियल के पदार्थो का अनुपात वॉल्यूम फ्रैक्शन इंडेक्स द्वारा नियंत्रित्र किया गया है।

चूंकि फंक्शनलली ग्रेडेड मटेरियल का प्रयोग अति उच्च तापमान पर किया जाता हैं अतः इन पैनलों में उच्च तापमान के कारण उत्पन कंपन का अध्ययन अनिवार्य हैं। अतः इस शोधकार्य में फंक्शनलली ग्रेडेड मटेरियल सैंडविच प्लेट और शैल पैनलों में उच्च तापमान द्वारा उत्पन कंपन का भी विश्लेषण किया गया हैं। तापमान को नियंत्र करने वाली अवकल समीकरण का हल सेंट्रल डिफरेंस एवम क्रेन्क -निकोल्सन तकनीक द्वारा प्राप्त किया गया है। तत्पश्चात गतिशील समीकरण का हल न्यूमॉर्क एवरेज एक्सीलेरेशन टाइम मार्चिंग तकनीक द्वारा प्राप्त करते हुए क्षणस्थायी विस्थापन और प्रतिबल की ग़णाना की गयी है।

वर्तमान लयेरवाइस सिद्धांत पर आधारित गणितीय संरचना की यथार्थता स्थापित करने के लिए इस संरचना द्वारा प्राप्त किये गये परिणामो की तुलना सवर्प्रथम मानक परिणामो से की गयी है । उपयुक्त जाली का चयन करने हेतु अभिसरण पर आधारित अध्ययन किया गया है जिसका प्रयोग लैमिनेटेड और सैंडविच प्लेट और शैल पैनलों के स्थिर और गतिशील व्यवहार का विश्लेषण करने हेतु किया गाया है । संरचना की यथार्थता सथापित करने के पश्चात् प्राचलिक अध्ययन किया गया है, ज्यामितीय, पदार्थो का अनुपात, तापमान, सैंडविच की संरचना का सैंडविच प्लेट और शैल पैनल के स्थिर और गतिशील व्यवहार का अध्यन किया गया है। यहाँ पर दर्शाया गया की प्रस्तावित लयेरवाइस सिद्धांत सहज, सरल एवम लैमिनेटेड और फंक्शनलली ग्रेडेड सैंडविच प्लेट और शैल पैनल का स्थिर और गतिशील व्यहवार की सटीक ग़णाना एवम विस्तृत श्रृंखला उद्धरणों को हल करने में शक्षम है।

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List of Symbols

Symbols used in present thesis have been listed here. The symbols that are not present in this list have been defined when they appear for first time.

x, y, z	*x, y* and *z* axes in rectangular coordinate system
a, b, h	Length, width and thickness of panel in rectangular plan
R_x, R_y	Radius of curvature in *x* and y directions
u, v, w	Displacements in *x, y* and *z* directions
i	Layer number
$z^{(i)}$	Local *z* axis of i^{th} layer
$u^{(i)}, v^{(i)}, w^{(i)}$	Displacement component along
u_0, v_0, w_0	Displacements of the middle plane along *x, y* and *z* directions
$\theta_x^{(i)}, \theta_y^{(i)}$	Rotations of normal to middle plane of i^{th} layer about *y* and *x* axes
$u_0^*, v_0^*, \theta_x^*, \theta_y^*$	Higher-order terms in the Taylor's series expansion
h_1, h_2, h_3	Thickness of bottom, middle and top layer
$\boldsymbol{\sigma}^{(i)}, \boldsymbol{\varepsilon}^{(i)}$	Stress and Strain vector corresponding to i^{th} layer
$\bar{\boldsymbol{\sigma}}^{(i)}, \bar{\boldsymbol{\varepsilon}}^{(i)}$	Stress resultants and mid-surface strain vector corresponding to i^{th} layer
$\mathbf{D}^{(i)}$	Rigidity matrix
$\mathbf{D}_\mathbf{m}^{(i)}, \mathbf{D}_\mathbf{s}^{(i)}$	Membrane and Shear rigidity matrix for any i^{th} layer
$\mathbf{D}_\mathbf{b}^{(i)}, \mathbf{D}_\mathbf{c}^{(i)}$	Bending, Membrane-Bending coupling for any i^{th} layer
$\mathbf{N}^{(i)}$	Shape function matrix of i^{th} layer
$\mathbf{B}^{(i)}$	Differential operator matrix of shape function of i^{th} layer
$\mathbf{I}^{(i)}$	Inertia matrix of i^{th} layer
$B^{(i)}$	Bulk modulus
$G^{(i)}$	Shear modulus
$E^{(i)}$	Young's modulus
$\nu^{(i)}$	Poisson's ratio
$\alpha^{(i)}$	Coefficient of thermal expansion
$K^{(i)}$	Coefficient of thermal conductivity
$P_{eff}^{(i)}$	Effective material property for any i^{th} layer
P_m, P_c	Elastic properties of pure metallic and ceramic
T	Temperature
T_0, T_u, T_l	Reference temperature, temperatures at top and bottom surfaces
C_v	Specific heat of material
Q_c	Heat flux
U_l	Strain energy due to linear strain

U_{nl}	Strain energy due to nonlinear strain
N_k	Shape function
T	Kinetic energy of the panel
$\mathbf{K}$	Global stiffness matrix of the panel
$\mathbf{K_G}$	Global geometric stiffness matrix of the panel
$\mathbf{M}$	Global mass matrix of the panel
W_{ext}	Total work done external applied transverse load *p*
$\mathbf{Q}_{th}$	Total thermal load vector
$\mathbf{d}$, $\dot{\mathbf{d}}$, $\ddot{\mathbf{d}}$	Displacement, Velocity and acceleration vectors
$\overline{\Omega}$	Non-dimensional natural frequency
E_1, E_2	Elastic modulus along longitudinal and transverse direction
G_{12}, G_{13}, G_{23}	Shear moduli
n	Volume fraction index

Organization of the Thesis

This thesis consists of eight chapters, Chapter 1 presents general introduction of problem, significance of present investigation and overview of methodology adopted. Chapter 2 focuses on review of available literature on static and dynamics analyses of laminated and functionally graded sandwich plates and shell panels. It also includes a review of problems related to thermally induced vibrations of functionally graded sandwich panels and gaps related to all considered areas. Scope and objectives of the present investigation are highlighted in Chapter 3. Detailed mathematical formulation of the problem is presented in Chapter 4. Results and discussion for problems related to static and dynamic analyses of laminated composite and sandwich plates and shell panels are discussed in Chapter 5. Chapter 6 comprises of results and discussion for static and dynamics analyses of functionally graded sandwich plates and shell panels. Behavior of FGM sandwich plate and shell panels subjected to thermal shock are discussed in Chapter 7. Conclusions based on discussion of results are presented in Chapter 8. List of references used in the present thesis are mentioned subsequently.