

BAYESIAN MODEL UPDATING USING COMPLEX MODAL DATA

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**DEPARTMENT OF CIVIL ENGINEERING
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BAYESIAN MODEL UPDATING USING COMPLEX MODAL DATA

by

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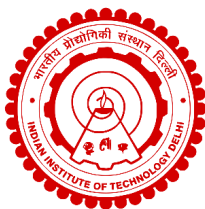
DEDICATION

*This thesis is dedicated to
my wife*

Jwan

and my sons

Zain and Yamen



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CERTIFICATE

This is to certify that the thesis titled “**Bayesian Model Updating Using Complex Modal Data,**” submitted by **Eamon Karim Henikish (Entry Number: 2019CEZ8793)**, to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under my supervision in the Department of Civil Engineering, Indian Institute of Technology Delhi. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Date:
Place: Delhi

Dr. Sahil Bansal

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ABSTRACT

There has been more than a century of history in modern civil engineering structures. These structures have undergone many effects in these times, altering their safety and health to an uncertain state that might be disastrous on some occasions. In this regard, Structural Health Monitoring (SHM) has emerged as an excellent choice for updating our information about these structures. However, the reference data from the construction time are mostly unavailable. This limits the use of data-based monitoring/detection of structural defects. Alternatively, model-based techniques such as model updating can be considered for applications of SHM and damage detection. Relying on model updating involves many uncertainties since modelling, in addition to collected data, encounters uncertainties. Bayesian model updating offers a solid approach to handling these uncertainties and provides plausibility measures for the best model amid a class of probable models representing the best fit to real data.

After twenty years of developing Bayesian model updating methods of civil and mechanical dynamical systems. Most methods solely use statistics such as Most Probable Values (MPVs) of modal parameters. This contradicts the nature of the Bayesian approach, which is strongly affected by higher moments information. In this thesis, modal data includes the MPVs and the variance/covariance matrix of modal parameters. Furthermore, in most cases in the literature, normal modes of vibration are used, and the damping parameters have been neglected. Damping existence usually alters the normal modal data to a complex form if distributed non-proportionally in the structure.

This study uses complex modal data, and a methodology is presented for Bayesian model updating using complex modal data without mode-matching or solution of the eigenvalue problem. The procedure is based on model reduction, where the full system

model is downsized to a smaller model corresponding to the measured Degrees of Freedom (DOFs). Establishing the model reduction-based method saves a lot of computational requirements. That is very important since the Bayesian approach is known to be computationally expensive.

The Bayesian approach faces the problem of finding the analytical solution for updating parameters. Usually, Markov Chain Monte Carlo (MCMC) algorithms are used to approximate the updating parameters' posterior Probability Density Function (PDF). This thesis presents a comprehensive MCMC solution, where a unique sampler called Metropolis-Within-Gibbs (MWG) is formulated to handle the high number of uncertain parameters. On top of that, an alternative method based on integrating system mode shapes is presented to account for the problems of locally identifiable cases.

One of the objectives of this work is to provide a comprehensive approach to be implemented with limited experimental tools for real experiments. A methodology that uses modal data from multiple setups is developed. The methodology requires the collection of partial mode shapes from multiple setups and does not need to form the global mode shape or scale the mode shape from a single setup.

Extensive validation of the proposed methods is done with various numerical problems with different numbers of DOFs and complexity. Also, different types of uncertainty in stiffness, mass, and damping characteristics are considered in these numerical examples. Furthermore, the approach is validated using two experimental models: a 3-DOF shear model and a laboratory model of the pedestrian bridge deck slab.

Keywords: Finite Element Modelling, Bayesian Model Updating, Linear Dynamical Systems, Complex Modes, Model Reduction, Markov Chain Monte Carlo, Uncertainty Quantification, Modal Data, Damage Detection.

अमूर्त

आधुनिक सिविल इंजीनियरिंग संरचनाओं का इतिहास एक सदी से भी ज़्यादा पुराना है। इन संरचनाओं पर इन समयों में कई प्रभाव पड़े हैं, जिससे उनकी सुरक्षा और स्वास्थ्य अनिश्चित अवस्था में बदल गया है जो कुछ मौकों पर विनाशकारी हो सकता है। इस संबंध में, संरचनात्मक स्वास्थ्य निगरानी (SHM) इन संरचनाओं के बारे में हमारी जानकारी को अपडेट करने के लिए एक बेहतरीन विकल्प के रूप में उभरी है। हालाँकि, निर्माण के समय का संदर्भ डेटा ज़्यादातर अनुपलब्ध है। यह संरचनात्मक दोषों की डेटा-आधारित निगरानी/पता लगाने के उपयोग को सीमित करता है। वैकल्पिक रूप से, मॉडल-आधारित तकनीक जैसे मॉडल अपडेटिंग को SHM और क्षति का पता लगाने के अनुप्रयोगों के लिए माना जा सकता है। मॉडल अपडेटिंग पर निर्भर होने से कई अनिश्चितताएँ शामिल हैं क्योंकि मॉडलिंग, एकत्रित डेटा के अलावा, अनिश्चितताओं का सामना करती है। बायेसियन मॉडल अपडेटिंग इन अनिश्चितताओं से निपटने के लिए एक ठोस दृष्टिकोण प्रदान करता है और वास्तविक डेटा के लिए सबसे उपयुक्त प्रतिनिधित्व करने वाले संभावित मॉडलों के एक वर्ग के बीच सर्वश्रेष्ठ मॉडल के लिए संभाव्यता उपाय प्रदान करता है। सिविल और मैकेनिकल डायनेमिक सिस्टम के बायेसियन मॉडल अपडेटिंग विधियों को विकसित करने के बीस वर्षों के बाद। अधिकांश विधियाँ केवल मोडल मापदंडों के सबसे संभावित मान (MPV) जैसे आँकड़ों का उपयोग करती हैं। यह बायेसियन दृष्टिकोण की प्रकृति का खंडन करता है, जो उच्चतर क्षणों की जानकारी से दृढ़ता से प्रभावित होता है। इस थीसिस में, मोडल डेटा में एमपीवी और मोडल मापदंडों के विचरण/सहप्रसरण मैट्रिक्स शामिल हैं। इसके अलावा, साहित्य में अधिकांश मामलों में, कंपन के सामान्य मोड का

उपयोग किया जाता है, और भिगोना मापदंडों की उपेक्षा की गई है। भिगोना अस्तित्व आमतौर पर सामान्य मोडल डेटा को एक जटिल रूप में बदल देता है यदि संरचना में गैर-आनुपातिक रूप से वितरित किया जाता है।

यह अध्ययन जटिल मोडल डेटा का उपयोग करता है, और मोड-मिलान या आइगेनवैल्यू समस्या के समाधान के बिना जटिल मोडल डेटा का उपयोग करके बायेसियन मॉडल अपडेट करने के लिए एक पद्धति प्रस्तुत की गई है। प्रक्रिया मॉडल कमी पर आधारित है, जहां पूर्ण सिस्टम मॉडल को मापी गई स्वतंत्रता की डिग्री (डीओएफ) के अनुरूप एक छोटे मॉडल में घटा दिया जाता है। मॉडल कमी-आधारित विधि की स्थापना से बहुत सारी कम्प्यूटेशनल आवश्यकताओं की बचत होती है। यह बहुत महत्वपूर्ण है क्योंकि बायेसियन दृष्टिकोण को कम्प्यूटेशनल रूप से महंगा माना जाता है।

बायेसियन दृष्टिकोण मापदंडों को अपडेट करने के लिए विश्लेषणात्मक समाधान खोजने की समस्या का सामना करता है। आमतौर पर, मार्कोव चेन मोंटे कार्लो (MCMC) एल्गोरिदम का उपयोग अपडेटिंग मापदंडों के पोस्टीरियर प्रोबेबिलिटी डेंसिटी फंक्शन (PDF) को अनुमानित करने के लिए किया जाता है। यह थीसिस एक व्यापक MCMC समाधान प्रस्तुत करती है, जहाँ मेट्रोपोलिस-विदिन-गिब्स (MWG) नामक एक अनूठा सैंपलर अनिश्चित मापदंडों की उच्च संख्या को संभालने के लिए तैयार किया गया है। इसके अलावा, स्थानीय रूप से पहचाने जाने योग्य मामलों की समस्याओं के लिए सिस्टम मोड आकृतियों को एकीकृत करने पर आधारित एक वैकल्पिक विधि प्रस्तुत की गई है। इस कार्य का एक उद्देश्य वास्तविक प्रयोगों के लिए सीमित प्रयोगात्मक उपकरणों के साथ कार्यान्वित किए जाने के लिए एक व्यापक दृष्टिकोण प्रदान करना है। एक कार्यप्रणाली विकसित की गई है जो कई सेटअपों से मोडल डेटा का उपयोग करती है। कार्यप्रणाली के लिए कई सेटअपों

से आंशिक मोड आकृतियों के संग्रह की आवश्यकता होती है और इसमें एकल सेटअप से वैश्विक मोड आकृति बनाने या मोड आकृति को स्केल करने की आवश्यकता नहीं होती है। प्रस्तावित विधियों का व्यापक सत्यापन विभिन्न संख्यात्मक समस्याओं के साथ विभिन्न संख्या में DOF और जटिलता के साथ किया जाता है। साथ ही, इन संख्यात्मक उदाहरणों में कठोरता, द्रव्यमान और भिगोना विशेषताओं में विभिन्न प्रकार की अनिश्चितता पर विचार किया जाता है। इसके अलावा, इस दृष्टिकोण को दो प्रयोगात्मक मॉडलों का उपयोग करके मान्य किया गया है: एक 3-डीओएफ शियर मॉडल और पैदल यात्री पुल डेक स्लैब का एक प्रयोगशाला मॉडल। कीवर्ड: परिमित तत्व मॉडलिंग, बेयसियन मॉडल अपडेटिंग, रैखिक गतिशील प्रणालियाँ, जटिल मोड, मॉडल रिडक्शन, मार्कोव चेन मॉन्टे कार्लो, अनिश्चितता परिमाणीकरण, मोडल डेटा, क्षति का पता लगाना।

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LIST OF ABBREVIATIONS AND SYMBOLS

DOF	Degree Of Freedom
FEM	Finite Element Model
MCMC	Markov Chain Monte Carlo
GS	Gibbs Sampler
MWG	Metropolis-Within-Gibbs
PDF	Probability Density Function
TMD	Tuned Mass Dampers
MPV	Most Probable Value
S.T. D	Standard Deviation
COV	Coefficient of Variation
TMCMC	Transitional Markov Chain Monte Carlo
SHM	Structural Health Monitoring
$E[.]$	Expectation
$V[.]$	Variance
θ	Vector of uncertain parameters
N_M	Number of measured modes
N_S	Number of setups
$D_{r,s}$	Set of measured data
$\hat{\omega}_r \in \mathbb{R}$	MPV of measured modal frequency
$\hat{\zeta}_r \in \mathbb{R}$	MPV of measured damping ratio
$\hat{\phi}_r \in \mathbb{C}^{n_m}$	MPV of mode shapes

$\mathbf{x}_{\phi,r}$	Appended form of complex mode shapes
$\hat{C}_{\omega,r}$	Variance of modal frequency
$\hat{C}_{\zeta,r}$	Variance of damping ratio
$\hat{\mathbf{C}}_{\phi,r}$	Covariance matrix of mode shapes
$\boldsymbol{\varepsilon}_r$	Prediction error function
$\boldsymbol{\sigma}_r^2$	Prediction error variance vector
$\boldsymbol{\omega}$	Vector of system modal frequencies
$\boldsymbol{\zeta}$	Vector of system damping ratio
$\mathbf{x}_{\phi}$	Vector of system mode shapes