OPTIMUM MULTIPLE FREQUENCY GENERATION
IN COLLISIONAL PLASMAS

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Thesis submitted to Indian Institute of Technology,Delhi

for partial fulfilment of the Degree of

Doctor of Philosophy

1973

ACKNOWLEDGEMENTS

I am deeply indebted to Professor M.S.Sodha and Dr. D.P.Tewari for their valuable guidance and kind encouragement during the course of the present work. Thanks are also due to Dr. R.L.Sawhney and Dr. V.K.Tripathi for helpful discussions, to Mr. T.N.Gupta for efficient typing of the manuscript. The financial support of C.S.I.R. (India) and E.S.S.A. (USA) is gratefully acknowledged.

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INTRODUCTION

The generation of high frequency electromagnetic waves (e.g. in the submillimeter range) is becoming more and more important on account of many applications of such waves. Most of the conventional generators do not operate at such high frequencies; the few which work, are very complex to design/fabricate and have poor efficiencies. The nonlinear properties of dielectrics and plasmas, however, provide an alternative mechanism of generating high frequency waves as harmonics of an intense electromagnetic wave. For over past two decades, considerable effort has been made to explore the feasibility of nonlinear multiple frequency generation in these media e.g. Margenau and Hartman (1948), Chiyoda (1967), Sodha, Sawhney and Sawhney (1970), Wetzel (1961) and Wetzel and Tang (1965), as a mechanism for operation of practical devices. The phenomenon of multiple frequency generation in plasmas can be understood physically as follows:-

In the presence of an alternating electric vector $\underline{E} = \underline{E}_0 e^{i\omega}$ the electrons of the plasma attain a drift velocity oscillating at a frequency ω. The oscillatory drift of carriers interacts with the magnetic field of the waves; thereby giving rise to a 2ω component in the electron drift velocity which results in the generation of second harmonic in the current density of the plasma.

Beside this relatively inefficient mechanism, collisions play an important role in the generation of harmonics in plasmas In the absence of an electric field a plasma is in thermal equilibrium and there is no exchange of energy between the electrons and the heavy scatterers. When an electric field is applied to a plasma, the electrons absorb energy from the field (due to their finite conductivity) and attain a temperature higher than that of heavier particles such that in steady state the energy gained from the field equals that transferred to heavier particles in collisions. As the electric field of an electromagnetic wave and the electron drift velocity are periodic in time, the power absorbed by the electrons from the wave has a time independent component as well as one oscillating at a frequency 2ω . In the steady state the rise in the time independent part of electron temperature (when $\nu^2 \ll \omega^2$) is given by

$$T_{e_0} - T_0 \simeq \alpha T_0 E E^* ,$$

the time dependent part (as a result of the solution of energy balance equation) is

$$T_{e2} \simeq \alpha T_0 \frac{2m\nu}{Mi\omega} E_0^2 e^{2i\omega t}$$

where

$$\alpha = \frac{e^2 M}{6 m^2 \omega^2 k_0 T_0}$$

-e and m are the electronic charge and mass respectively, M is the mass of the scatterers, ν is the electron collision frequency, k_0 is the Boltzmann constant and T_0 is the equilibrium temperature of electrons. The field dependent temperature of electrons modulates the electron collision frequency ν which in general is a function of electron temperature. Thus ν can be written as

$$\nu = \nu_0 + \nu_2 e^{i2\omega t} \tag{3}$$

Due to this modulation of ν, the complex conductivity σ, given by

$$\sigma = \frac{Ne^2}{m(\nu+i\omega)}$$

of the plasma acquires an oscillating component (at frequency 2ω) the current density $\underline{J} = \sigma \underline{E}$ has a component oscillating with frequency 3ω resulting in the generation of third harmonic. It is also clear from the above phenomenological argument that the second and other even harmonics cannot be generated by this mechanism unless an additional d.c. field is applied to the plasma or when the plasma is non-uniform; this is on account of the d.c. conductivity and diffusion coefficient having an oscillating component of frequency 2ω due to modulated electron temperature.

It must be mentioned here that the phenomenological argument is only qualitatively true. Strictly speaking, the assumption $\nu < \omega$ means that the energy relaxation time is much larger than the wave period and hence the concept of an oscillating temperature (varying as $e^{2i\omega t}$) is only marginally helpful. To obtain reasonable quantitative estimates of harmonic generation one should solve the Boltzmann equation for the electron velocity distribution function and hence obtain expressions for the current density; this expression should be substituted in the wave equation to obtain an expression for the electric vector of the generated harmonics in specific cases. Some workers in recent years have followed this method to study the generation of second and third harmonics in plasmas. An important conclusion of these investigations is that in the Cartesian tensor expansion of the electron velocity distribution function in the velocity space, proper care must be exercised in accounting, for the second order tensor $\underline{\underline{f_2}}$, whose contribution to the harmonic part in has been found to be of the same order as that arising from Gupta has extended this treatment to study generation of harmonics in a magnetoplasma. But their omission of off-diagonal terms of $\underline{\underline{f_2}}$ has lead to somewhat erroneous results; one of them is the prediction of third harmonic generation by a single circularly polarized mode.

In the proposed thesis, the author has presented a self consistent kinetic treatment of the phenomenon of harmonic generation in a magneto plasma. The collisional mechanism has been held responsible for the nonlinear mixing. The presence of a static magnetic field in the plasma enhances the harmonic output due to cyclotron effects at gyro-resonance. In order to consider the practical aspects of the problem the plasma has been considered in the form of a slab of finite thickness, the introduction of boundaries introduces certain dimensional resonances in the harmonic output, which can be explained physically as follows:

Consider a strong electromagnetic wave incident normally on a plasma slab. If the thickness of the slab is zero no harmonics are generated as there is no nonlinear medium to interact with. Further, if the thickness is infinite all the harmonics will be absorbed in the plasma. Thus there should be some optimum thickness for which the transmitted harmonic is of maximum amplitude. In addition to this obvious optimizing effect, the mutual interaction of forward and backward propagating waves inside the plasma gives rise to several interference effects. In order to generate even harmonics either the plasma has to be subjected to a d.c. electric field or have a non-uniformity in the electron concentration. The entire work is divided into the following chapters:-

I. Optimum Generation of Third Harmonic in Homogeneous Magnetoplasmas.

II. Optimum Generation of Second Harmonic in Inhomogeneous Magnetoplasmas.

III. Optimum Second Harmonic Generation in Homogeneous Magnetoplasma in the presence of Static electric field.

The following publications have resulted from this work:-

1. Optimum Third Harmonic Generation in Magnetoplasmas Plasma Physics, 13, 373-86 (1971).

2. Optimum Second Harmonic Generation in a Inhomogeneous Magnetoplasma, Plasma Physics, In Press (1973).

3. Optimum Second Harmonic Generation in a Magnetoplasma in presence of a dc Electric Field, Plasma Physics, communicated (1973).

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