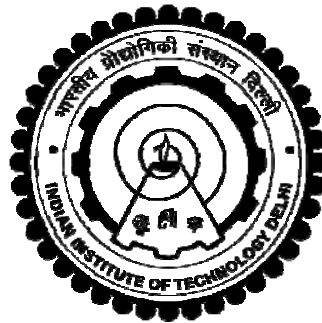


**TOWARDS A FRAMEWORK FOR WAVELET
BASED MULTI-CARRIER TRANSMISSION
SYSTEMS**

SUJATA SENGAR



**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
HAUZ KHAS, NEW DELHI 110016**

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by

SUJATA SENGAR

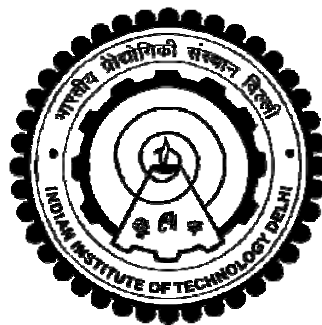
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Submitted

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Certificate

This is to certify that the thesis entitled “**Towards a Framework for Wavelet based Multi-Carrier Transmission Systems**” being submitted by Ms. Sujata Sen-
gar to the Department of Electrical Engineering, Indian Institute of Technology,
Delhi is the record of the bonafide research work carried out by her. She has
worked under our supervision and guidance during the period August 2006 to
July 2013. She has fulfilled all the requirements for submission of the thesis
which has reached the requisite standard.

The results contained in the thesis have not been submitted either in part or in
full to any other university or institute for the award of any degree or diploma.

Dr. Surendra Prasad

Professor

Department of Electrical Engineering,
Indian Institute of Technology Delhi

Dr. Shiv Dutt Joshi

Professor

Department of Electrical Engineering,
Indian Institute of Technology Delhi

Date:

New Delhi

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Abstract

The transition from single carrier to multi-carrier transmissions has resulted in a significant improvement in the performance of communication over fading channels. The Orthogonal Frequency Division Multiplexing (or the OFDM) is the most popular multi-carrier scheme deployed in a range of communication applications. The popularity of these schemes in high data rate applications, over the more traditional single carrier schemes may be attributed largely to the simplicity of the resulting transceiver structures, and the availability of fast computational algorithms (like the FFT etc.) for the realization of the filter banks associated with these schemes. To this day, virtually no commercial application uses a multi-carrier modulation based on anything other than the OFDM or its different avatars (like the DMT in DSL technologies). However, it has been felt for sometime now that a more general framework for multi-carrier schemes can be built around the multi-rate filter bank transceivers. The multi-rate theory of filter banks and wavelet representations fit well into such a multi-carrier framework. This has led to the use of wavelet theory as a good platform on which multi-carrier waveform bases can be built. Fractal modulation, which has received some attention in the literature, is one example of a wavelet-based communication. In the context of Fractal modulation, it has been shown that a frequency selective fading (FSF) channel introduces a random gain for each scale, whose variance is scale dependent, and increases exponentially with scale.

The main objective of this thesis is to explore the feasibility of using wavelet

functions as carriers for wireless communication channels in a more general setting, and as an alternative to conventional multi-carrier schemes like OFDM. To this end, we take up a systematic study of different types of wavelet modulations possible, their associated transceiver architectures, etc., and attempt to develop an insight into their properties as relevant to communication over fading dispersive channels. We also consider optimization of their performance through power and bit loading algorithms, as pertinent to the wavelet framework. Some of the main results of the thesis are briefly described in the following.

We first consider the development of a multi-rate filter bank transceiver structure for a general form of wavelet modulation. It is shown that the computational complexity of a wavelet transceiver is comparable with or even better than that of IFFT/FFT based OFDM systems. Another interesting perspective that emerges from this study is that wavelet modulation can be interpreted in two different, but equivalent ways. In one, it is essentially seen as a method of mapping data symbols onto wavelet functions of different scales and translates in some appropriate manner. In the other, alternative view it can also be regarded as a matrix (or wavelet) pre-coder whose output is embedded onto different translates of the scaling function at different scales. Both these perspectives are useful in studying the behaviour of wavelet modulation in different environments.

We next consider a class of wavelet modulations, which can provide a graceful trade-off between spectral efficiency and scale diversity, in a frequency selective fading environment. The scale diversity is controlled by the manner in which data is mapped onto different scales and translates of the underlying wavelet. Three schemes, that allow different orders of scale diversity are studied, of which the previously studied fractal modulation is shown to be a special case.

Each of the three formats (referred to as Cases I, II and III, respectively) is first studied for its symbol error rate performance, under the assumption that the receiver has perfect channel state information. The thesis attempts to address the

problem of minimizing the bit error rate by exploiting the fact that statistics of the scale gains are known apriori. To this end, two strategies have been explored. First, an optimum power allocation problem across scales under a total power constraint is formulated and solved in a closed form. In the second approach, we consider the problem of bit loading of different scales for a given transmit power. For this approach, further optimization is undertaken by optimum power allocation across those scales, which have similar bit loading. The two approaches provide some interesting design insights into the wavelet based multi-carrier systems in terms of their diversity and coding gain advantages and spectral efficiencies.

The performance of all three cases of wavelet modulation have also been compared with OFDM on the basis of spectral efficiency for a desired bit error rate and the maximum achievable spectral efficiency (as ascertained from average achievable rate calculations). Overall, the performance of a form of Wavelet Modulation (Case III) with bit loading and power control in a frequency selective channel is shown to offer the best bit-error rate performance for lower end of desired spectral efficiencies. However, another form referred to as Case I, appears to be best suited for applications demanding higher spectral efficiencies.

While the Case III of Wavelet modulation offers better BER performance, it is also seen to suffer from a loss of spectral efficiency due to explicit mapping of data to multiple scales. We show that it is possible to compensate this loss via the use of Multi-wavelets by increasing the available degrees of freedom by deploying a set of 'R' different generating wavelets that can be transmitted in the same bandwidth, and that can carry independent data streams, thus yielding a higher spectral efficiency. We also explore the potential for introducing space-scale coding with a view to achieve higher diversity orders in the framework of multi wavelet models.

Finally, we invoke the alternative interpretation of wavelet modulation as a wavelet pre-coder to study its performance in a time-selective fading channel. It is shown that the fact that symbols at various scales are spread over overlapping

time/frequency bands, can be used to advantage to design a successive cancellation receiver structure, which can yield a diversity of order two in a time-selective fading channel with known channel state information at the receiver.

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