DISCRETE KIRCHHOFF FINITE ELEMENTS FOR PIEZOELECTRIC HYBRID PLATES BASED ON SMEARED AND ZIGZAG THIRD ORDER THEORIES

by

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Certificate

*This is to certify that the thesis entitled "**Discrete Kirchhoff Finite Elements for Piezo-electric Hybrid Plates Based on Smeared and Zigzag Third Order Theories**" being submitted by Kulkarni Shripad Dattatraya to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.*

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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S.D. Kulkarni

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Abstract

Two new improved discrete Kirchhoff four-node quadrilateral elements based on the third-order zigzag theory (ZIGT) and the smeared third order theory (TOT) are developed for the dynamic analysis of elastic composite and sandwich plates. The elements have seven degrees of freedom (DOF) per node, namely, the three displacements, two rotations and two transverse shear strain components at the mid-surface. The usual requirement of C^1 continuity of interpolation functions of the deflection in the ZIGT and TOT is circumvented by employing the improved discrete Kirchhoff constraint technique (IDKQ), which was originally proposed for the bending of isotropic thin Kirchhoff plates and have been used herein for shear deformable composite plates for the first time. The consistent mass matrix, the element stiffness matrix and the load vector are derived using the Hamilton' principle. It is revealed from the numerical studies that the IDKQ interpolation of deflection yields faster convergence for response entities than the discrete Kirchhoff quadrilateral (DKQ) interpolation. The elements do not suffer from the shear locking problem. They are very little sensitive to element distortion for the deflection and fundamental frequency, and are moderately sensitive for the stresses and frequencies of higher modes. The finite element (FE) formulation and the computer program are validated by comparing the results for the static and free vibration response of simply-supported plates with the analytical Navier solutions of the ZIGT and the TOT. Comparison of the present results for static response and natural frequencies of composite and sandwich plates with those using other available elements based on zigzag theories and the TOT establishes the superiority of the present quadrilateral elements in respect of accuracy and computational efficiency. The accuracy of the ZIGT theory is assessed for plates with various laminate configurations, shapes, boundary conditions and aspect ratio with the exact three-dimensional (3D) elasticity solution for simply-supported plates and with the converged 3D FE solution obtained using ABAQUS for other boundary conditions. The comparison also establishes the superiority of the ZIGT theory over the smeared TOT having the same number of DOF.

An efficient quadrilateral element, DKIZIGT, is developed next based on an improved zigzag theory (IZIGT) for the dynamic analysis of hybrid plates with electroded piezoelectric sensors and actuators. The theory considers a third order zigzag approximation for inplane displacements, a layerwise quadratic approximation for the electric potential, and a layerwise variation of the deflection to account for the piezoelectric transverse normal strain. The conditions on transverse shear stresses at the interfaces and at the top and bottom are satisfied exactly in presence of electric loading to express the displacement field in terms of only five displacement variables. Like the elastic plate elements, this element too has four physical nodes with seven kinetic DOF per node, and the deflection is interpolated using the IDKQ technique which is found to be superior to the DKQ technique for the hybrid plates too. By introducing an electric node in the element for the electric potentials of the electroded piezoelectric surfaces, the equipotential condition of such surfaces is modelled very efficiently. In a novel concept, the electric potential DOF corresponding to the quadratic component of the electric potential distribution are associated with the physical nodes to allow for the inplane electric field induced due to direct piezoelectric effect. The element mass matrix, electromechanical stiffness matrix and the electromechanical load vector are obtained using extended Hamilton's principle. The element based on the coupled improved third order theory (ITOT) is developed as a special case. Both the elements are shear locking free. It is illustrated that it is possible to apply a nonuniform potential distribution over a piezoelectric actuator surface by segmenting the surface in a number of small equipotential electrodes, which yields deflection and stresses that are very close to the continuous distribution case. The number of segments in the electroded surfaces in open circuit condition can have appreciable effect on the deflection and natural frequencies of hybrid plates. The present results for the static and free vibration response for a variety of bimorph, hybrid composite and sandwich plates, under mechanical and electric potential loads are compared with 3D analytical and FE solutions, and those of other available elements based on efficient zigzag theories. It is concluded the present DKIZIGT element yields accurate results for active, sensory and combined active-sensory dynamic response of moderately thick to thin hybrid composite and sandwich plates for all kinds of lay-ups, shapes, mechanical boundary conditions and electro-mechanical loading. The element is superior to other available elements based on efficient layerwise theories in respect of accuracy, robustness and computational efficiency. The present elements are suitable for general purpose FE programming.

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