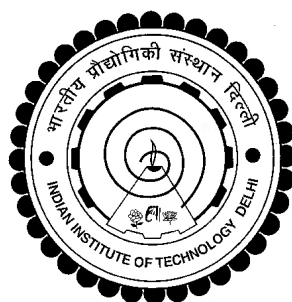


**COMPACT FINITE DIFFERENCE METHODS WITH ERROR
ANALYSIS FOR PROBLEMS ARISING IN OPTION PRICING**

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**DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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ANALYSIS FOR PROBLEMS ARISING IN OPTION PRICING**

by

KULDIP SINGH PATEL

Department of Mathematics

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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**Dedicated to
My Family**

Certificate

This is to certify that the thesis entitled “**Compact finite difference methods with error analysis for problems arising in option pricing**” submitted by “**Mr. Kuldip Singh Patel**” to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for award of any degree or diploma.

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Abstract

Over the past few decades, problem of option pricing has been a major development in mathematical finance. Mathematical modelling of option pricing problems mainly leads to partial differential equations (PDEs), partial integro-differential equations (PIDEs), and linear complementarity problems (LCPs). Because of their complex nature, it is not always possible to obtain the analytical solutions of these PDEs, PIDEs, and LCPs. Therefore, accurate and efficient numerical methods are required to solve them numerically.

Finite difference schemes are one of the oldest and well established techniques for numerical solution of PDEs. One can develop high-order accurate finite difference schemes by increasing computational stencil, but it complicates the implementation of boundary conditions. Furthermore, application of high-order finite difference schemes sometimes suffer from restrictive stability conditions and spurious oscillations may also be present in the numerical solutions. Therefore, finite difference schemes have been developed using compact stencils (commonly known as compact schemes) at the expense of some complication in their evaluation.

In order to have a better understanding of compact approximations, we developed MATLAB routines to compute derivative approximations of arbitrary functions. High-order compact approximations for first, second, third, and fourth derivatives are obtained for functions with periodic, Dirichlet, and Neumann boundary conditions. Moreover, compact approximation for partial derivative approximations of functions in two variables

are also discussed.

In this thesis, compact schemes are proposed for solving PDEs, PIDEs and LCPs arising in option pricing. Novelty of proposed compact schemes is that they do not require original equations as an auxiliary equations unlike other compact schemes. In proposed compact schemes, second derivative is expressed in terms of unknowns itself and their first derivative approximations, thereby allowing us to obtain a tridiagonal system of linear equations for fully discrete problems. Fourier analysis of differencing error for proposed compact approximation of second derivative is presented. It is observed that proposed compact approximation for second derivative has better resolution characteristics as compared to other difference approximations. We started with numerical approximation to the solution of one-dimensional linear convection-diffusion equation. Consistency and stability of fully discrete problem are proved and it is shown that proposed compact scheme is fourth order accurate.

Nevertheless, compact approximations are not customary tools for option pricing problems because initial conditions for these problems are always non-smooth. As a result, it will affect the convergence rate of proposed compact schemes. Several approaches, for example, co-ordinate transformation and local mesh refinement have been considered in literature to achieve high-order convergence rate even for non-smooth initial conditions. These approaches suffer with certain drawbacks, for example, it may not be possible to define a coordinate transformation for all PDEs, PIDEs and LCPs. Further, manual inclusion of grid points near singularity to accomplish local mesh refinement becomes tedious in certain cases. In order to avoid these limitations, smoothing operator are employed to smooth the initial conditions which helps to achieve high-order convergence rate. Moreover, an efficient numerical method is developed for solving PDEs using wavelet which does not require manual inclusion of grid points near singularity.

In order to illustrate the applicability of proposed compact scheme in option pricing, we adopted the compact scheme to solve Black-Scholes PDE for pricing European options. It is observed that proposed compact scheme is only second order accurate with original (non-smooth) initial condition and fourth order accuracy is achieved after em-

ploying smoothing operator to initial condition. As an application of convection-diffusion equations, proposed compact scheme is adopted to solve PDE governing price of Asian options. It is shown that proposed compact scheme is accurate for Asian option PDE also.

We considered compact schemes based on different temporal semi-discretizations for solving PIDE and LCP governing European and American option prices respectively under jump-diffusion models. Consistency and stability of proposed compact schemes are also proved. As an extension to jump-diffusion models, we proposed compact schemes to solve coupled PIDEs and LCPs for pricing European and American options under regime-switching jump-diffusion models (RSJDM). Numerical illustrations are presented to demonstrate the accuracy and efficiency of proposed compact schemes.

Considering the fact that local mesh refinement helps to achieve high-order convergence rate, we proposed a wavelet optimized compact scheme (WOCS) to solve PDEs using diffusion wavelet. Solution of PDEs using WOCS is obtained on adaptive grid, which implies more grid points are added near singularity and grid points are removed where solution is smooth. Thus, proposed WOCS automatically takes care of local mesh refinement and it is shown that proposed WOCS is extremely efficient. As an application in option pricing, Black-Scholes PDE for barrier options is solved using proposed WOCS.

In order to validate the applicability of compact approximations for fractional derivatives, we derived a fourth order compact approximation for space fractional Riemann-Liouville derivative. Using this approximation, fourth-order compact schemes are proposed to solve space fractional convection-diffusion equations in one and two dimensions. As an application in option pricing, space fractional Black-Scholes equation is solved using proposed compact scheme. Fourier analysis of differencing errors for fractional derivatives is presented which may be considered to illustrate resolution characteristics of compact approximations rather than its truncation errors.

सार

पिछले कुछ दशकों में, विकल्प मूल्य निर्धारण की समस्या गणितीय वित्त में एक बड़ा विकास रहा है। विकल्प मूल्य निर्धारण समस्याओं का गणितीय मॉडलिंग मुख्य रूप से आंशिक अंतर समीकरण (पीडीई), आंशिकअभिन्न-अंतरसमीकरण (पीआईडीएस), और रैखिक पूरकता समस्याओं (एलसीपी) की ओर जाता है। उनकी जटिल प्रकृति के कारण, इनपीडीई, पीआईडीएस और एलसीपी के विश्लेषणात्मक समाधान प्राप्त करना हमेशा संभव नहीं होता है। इसलिए, उन्हें संख्यात्मक रूप से हल करने के लिए सटीक और कुशल संख्यात्मक विधियों की आवश्यकता होती है।

परिमित अंतरयोजनाएं पीडीई के संख्यात्मक समाधान के लिए सबसे पुरानी और अच्छी तरह से स्थापित तकनीकों में से एक हैं। कम्प्यूटेशनल स्टैंसिल को बढ़ाकर उच्च-आदेश सटीक सीमित अंतर योजनाएं विकसित कर सकती हैं, लेकिन यह सीमा की स्थिति के कार्यान्वयन को जटिल बनाती है। इसके अलावा, उच्च-आदेश सीमित अंतरयोजनाओं का उपयोग कभी-कभी प्रतिबंधित स्थिरता की स्थिति से ग्रस्त होता है और नकली परिसंचरण संख्यात्मक समाधानों में भी उपस्थित हो सकता है। इसलिए, सीमित मूल्यांकन योजनाओं को उनके मूल्यांकन में कुछ जटिलताओं के खर्च पर कॉम्पैक्ट स्टैंसिल (आमतौर पर कॉम्पैक्ट स्कीम के रूप में जाना जाता है) का उपयोग करके विकसित किया गया है।

कॉम्पैक्ट अनुमानों की बेहतर समझ रखने के लिए, हमने मनमाने ढंग से कार्यों के व्युत्पन्न अनुमानों की गणना करने के लिए मैटलैब रूटीन विकसित किए। आवधिक, डिरिचलेट और न्यूमैन सीमा स्थितियों के साथ कार्यों के लिए पहले, दूसरे, तीसरे, और चौथे डेरिवेटिव के लिए उच्च-आदेश कॉम्पैक्ट अनुमान प्राप्त किए जाते हैं। इसके अलावा, दो चरों में कार्यों के आंशिक व्युत्पन्न अनुमानों के लिए कॉम्पैक्ट सन्निकटन पर भी चर्चा की जाती है।

इस थीसिस में, विकल्प मूल्य निर्धारण में उत्पन्न पीडीई, पीआईडी और एलसीपी को हल करने के लिए कॉम्पैक्ट योजनाएं प्रस्तावित की जाती हैं। प्रस्तावित कॉम्पैक्ट योजनाओं की नवीनता यह है कि उन्हें मूल समीकरणों को अन्य कॉम्पैक्ट योजनाओं के विपरीत सहायक समीकरणों की आवश्यकता नहीं होती है। प्रस्तावित कॉम्पैक्ट योजनाओं में, दूसरा व्युत्पन्न अज्ञात और उनके पहले व्युत्पन्न अनुमानों के संदर्भ में व्यक्त किया जाता है, जिससे हमें पूरी तरह से अलग समस्याओं के लिए रैखिक समीकरणों की त्रिभुज प्रणाली प्राप्त करने की इजाजत मिलती है। दूसरे व्युत्पन्न के प्रस्तावित कॉम्पैक्ट अनुमान के लिए त्रुटि का फूरियर विश्लेषण प्रस्तुत किया गया है। यह देखा गया है कि दूसरे व्युत्पन्न के लिए प्रस्तावित

कॉम्पैक्ट सन्निकटन में अन्य अंतर अनुमानों की तुलना में बेहतर रिज़ॉल्यूशन विशेषताएं हैं। हमने एक-आयामी रैखिक संवहन-प्रसार समीकरण के समाधान के लिए संख्यात्मक अनुमान के साथ शुरू किया। पूरी तरह से अलग समस्या की सुसंगतता और स्थिरता साबित होती है और यह दिखाया जाता है कि प्रस्तावित कॉम्पैक्ट योजना चौथा आदेश सटीक है।

फिरभी, कॉम्पैक्ट अनुमान विकल्प मूल्य निर्धारण समस्याओं के लिए पारंपरिक उपकरण नहीं हैं क्योंकि इन समस्याओं के लिए प्रारंभिक स्थितियां हमेशा गैर-चिकनी होती हैं। नतीजतन, यह प्रस्तावित कॉम्पैक्ट योजनाओं की अभिसरण दर को प्रभावित करेगा। उदाहरण के लिए, कई दृष्टिकोण, गैर-चिकनी प्रारंभिक स्थितियों के लिए भी उच्च-आदेश अभिसरण दर प्राप्त करने के लिए साहित्य में परिवर्तन और स्थानीय जाल परिशोधन को समन्वयित किया गया है। ये दृष्टिकोण कुछ कमियों के साथ पीड़ित हैं, उदाहरण के लिए, सभीपीडीई, पीआईडी और एलसीपी के लिए समन्वय परिवर्तन को परिभाषित करना संभव नहीं हो सकता है। आगे, मैन्युअल समावेशन स्थानीय जाल पुनः जुर्माना पूरा करने के लिए एकवचन के पास ग्रिड अंक के कुछ मामलों में थकाऊ हो जाता है। इन सीमाओं से बचने के लिए, प्रारंभिक स्थितियों को सुचारू बनाने के लिए चिकनाई ऑपरेटर को नियोजित किया जाता है जो उच्च-आदेश अभिसरण दर प्राप्त करने में मदद करता है। इसके अलावा, वेवलेट का उपयोग करके पीडीई को हल करने के लिए एक कुशल संख्यात्मक विधि विकसित की जाती है जिसे एकवचन के पास ग्रिडबिंदुओं के मैन्युअल समावेशन की आवश्यकता नहीं होती है।

विकल्प मूल्य निर्धारण में प्रस्तावित कॉम्पैक्ट योजना की प्रयोज्यता को स्पष्ट करने के लिए, हमने यूरोपीय विकल्पों के मूल्य के लिए ब्लैक-स्कॉल्स पीडीई को हल करने के लिए कॉम्पैक्ट योजना अपनाई। यह देखा गया है कि प्रस्तावित कॉम्पैक्ट योजना मूल (गैर-चिकनी) प्रारंभिक स्थिति के साथ केवल दूसरा ऑर्डर सटीक है और प्रारंभिक स्थिति में स्मूथिंग ऑपरेटर को नियोजित करने के बाद चौथी ऑर्डर सटीकता प्राप्त की जाती है। संवहन-प्रसार समीकरणों के एक आवेदन के रूप में, एशियाई विकल्पों के पीडीई शासित मूल्य को हल करने के लिए प्रस्तावित कॉम्पैक्ट योजना अपनाई जाती है। यह दिखाया गया है कि प्रस्तावित कॉम्पैक्ट योजना एशियाई विकल्प पीडीई के लिए भी सटीक है।

हमने कूद-प्रसार मॉडलों के तहत क्रमशः यूरोपीय और अमेरिकी विकल्प की कीमतों को नियंत्रित करने के लिए पीआईडी और एलसीपी को हल करने के लिए विभिन्न अस्थायी सेम-विघटन के आधार पर कॉम्पैक्ट योजनाएं मानीं। प्रस्तावित कॉम्पैक्ट योजनाओं की सुसंगतता और स्थिरता भी साबित हुई है। कूद-प्रसार मॉडलों के विस्तार के रूप में, हमने युग्मित पीआईडी और एलसीपी को नियम-स्विचिंग जंप-प्रसार मॉडलों (आरएसजेडीएम) के तहत यूरोपीय और अमेरिकी विकल्पों के मूल्य निर्धारण के लिए कॉम्पैक्ट योजनाओं का प्रस्ताव

दिया। प्रस्तावित कॉम्पैक्ट योजनाओं की सटीकता और दक्षता का प्रदर्शन करने के लिए संख्यात्मक चित्र प्रस्तुत किए गए हैं।

इस तथ्य को ध्यान में रखते हुए कि स्थानीय जाल परिशोधन उच्च-आदेश अभिसरण दर प्राप्त करने में मदद करता है, हमने प्रसार लहरों का उपयोग करके पीडीई को हल करने के लिए एक वेवलेट अनुकूलित कॉम्पैक्ट योजना (डब्ल्यूओसीएस) का प्रस्ताव दिया।

डब्ल्यूओसीएस का उपयोग करते हुए पीडीई का समाधान अनुकूली ग्रिड पर प्राप्त होता है, जिसका अर्थ है कि एकवचन के पास अधिक ग्रिड अंक जोड़े जाते हैं और ग्रिड पॉइंट हटा दिए जाते हैं जहां समाधान चिकनी होता है। इस प्रकार, प्रस्तावित डब्ल्यूओसीएस स्वचालित रूप से स्थानीय जाल परिशोधन का ख्याल रखता है और यह दिखाया गया है कि प्रस्तावित डब्ल्यूओसीएस बेहद कुशल है। विकल्प मूल्य निर्धारण में एक आवेदन के रूप में, बाधा विकल्पों के लिए ब्लैक-स्कॉल्स पीडीई प्रस्तावित डब्ल्यूओसीएस का उपयोग कर के हल किया जाता है।

आंशिक डेरिवेटिव्स के लिए कॉम्पैक्ट अनुमानों की प्रयोज्यता को प्रमाणित करने के लिए, हमने अंतरिक्ष आंशिक रिमैन-लिओविले व्युत्पन्न के लिए चौथा आदेश कॉम्पैक्ट अनुमान लगाया। इस सन्निकटन का उपयोग करते हुए, चौथे क्रम कॉम्पैक्ट योजनाओं को एक और दो आयामों में अंतरिक्ष आंशिक संवहन-प्रसार समीकरणों को हल करने का प्रस्ताव है। विकल्प मूल्य निर्धारण में एक आवेदन के रूप में, स्पेस आंशिक ब्लैक-स्कॉल्स समीकरण प्रस्तावित कॉम्पैक्ट योजना का उपयोग कर के हल किया जाता है। आंशिक डेरिवेटिव्स के लिए त्रुटियों का फूरियर विश्लेषण प्रस्तुत किया जाता है जिसे इसकी छंटनी त्रुटियों के बजाय कॉम्पैक्ट अनुमानों की रिज़ॉल्यूशन विशेषताओं को चित्रित करने के लिए माना जा सकता है।

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