ON DISCRETIZATION OF CONTINUOUS-TIME DYNAMICAL SYSTEMS

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"If you cannot do great things, do small things in a great way"
-Napoleon Hill

I dedicate this work, to my teachers, friends and family members.

Certificate

This is to certify that the thesis entitled **"On Discretization of Continuous-time Dynamical Systems"**, submitted by **Gagan Deep Meena** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the original, bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

Dr. S. Janardhanan,
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Abstract

This thesis presents an in-depth documentation of new techniques and their applications in the area of discretization of continuous-time dynamical systems. The concept of exact discretization has been explored that for continuous-time nonlinear and linear time-varying systems. Exact discretization means the state trajectories of a given continuous-time system should match exactly with state trajectories of the discretized system at each sampling instant. The detailed literature review about the developments made in the field of discretization techniques has been presented after a basic introduction on discretization. An explanation of existing canonical techniques, like Euler and Taylor methods, has been given. Taylor methods are based on Taylor series expansion which results in the exact discretization of continuous-time systems. Linear time-invariant systems are exactly discretizable by Taylor methods. The extension of this class of exactly discretizable systems has been done. A new formulation has been proposed which uses Lie algebraic operators in Taylor series expansion to obtain the discrete version of given continuous-time system. The proposed formulation is the generalised version of an earlier proposed theory. The method has been tested on three different standard nonlinear oscillator models: Van der Pol Oscillator, Duffing Oscillator, and FitzHugh Nagumo model. A comparative analysis with existing canonical methods has been done which shows the effectiveness of proposed method over existing methods for discretization. Usually, Taylor series based methods are used for exact discretization of the continuous-time systems but they have been found to be ineffective in case of linear time varying (LTV) systems. The reason for avoiding Taylor based methods for exact discretization has been discussed and an alternative method has been suggested. This is an integration approach based method for obtaining the exact discrete version. Parameter-based analysis has been done with the help of Matlab simulations for all proposed techniques with the support of different numerical examples.

Chapter 1 is the introduction to the thesis which explains the motivation and builds the background for subsequent chapters. Chapter 2 contains a detailed review of important works in the field of discretization with emphasis on nonlinear dynamical systems. Chapter 3 deals with the issue of discretization of continuous time dynamical systems, it presents existing method of exact discretization fro linear systems and extends it to nonlinear systems. Chapter 4 proposes

a new theory for discretizing the nonlinear system by using Lie algebraic operators and in Chapter 5 the applicability of Taylor-Lie formulation to more classical nonlinear systems is analyzed. Chapter 6 deals with LTV systems and proposes the strategy for their discretization. Finally, in the concluding chapter, a summary of the thesis is presented and future directions of research are identified.

सार

यह शोध प्रबंध निरंतर समय क्रियाशील प्रणालियों के विघटन के क्षेत्र में नई तकनीकों और उनके अनुप्रयोगों का गहन दस्तावेज प्रस्तुत करता है। नॉनलीनियर और लीनियर टाइम वेरिंग प्रणालियों के लिए सटीक विघटन की अवधारणा का अध्ययन किया गया है । सटीक विघटन का मतलब है कि दिए गए निरंतर समय प्रणाली के स्तिथि प्रतिक से विघटित प्रणाली के स्तिथि प्रतिक का सम्पूर्ण मेल हो | विघटनकारी तकनीकों के क्षेत्र में किए गए विकास के बारे में विस्तृत साहित्य समीक्षा को विघटन पर बुनियादी परिचय के बाद प्रस्तुत किया गया है। ऑयलर और टेलर विधियों जैसे मौजूदा विहित तकनीकों का एक स्पष्टीकरण दिया गया है। टेलर विधियां टेलर श्रृंखला विस्तार पर आधारित होती हैं जिसके परिणामस्वरूप निरंतर समय प्रणाली का सटीक विघटन होता है। लीनियर टाइम इनवेरिएंट प्रणाली टेलर विधियों द्वारा सटीक विघटनकारी हैं। सटीक विघटनकारी प्रणालियों के इस वर्ग का विस्तार किया गया है। एक नया सूत्रीकरण किया गया है जो टेलर श्रृंखला विस्तार में ली अल्जेब्रिक ऑपरेटरों का उपयोग, निरंतर समय प्रणाली के विघटित संस्करण को प्राप्त करने के लिए करता है। प्रस्तावित सूत्रीकरण पहले से प्रस्तावित सिद्धांत का व्यापकन है। विधि का परीक्षण तीन विभिन्न मानक नॉनलीनियर ऑसीलेटर मॉडलस, वैन डर पोल ऑसीलेटर, डफिंग ऑसीलेटर, और फिट्जहुग नागुमो मॉडल पर किया गया है। मौजूदा विहित तरीकों के साथ एक तुलनात्मक विश्लेषण किया गया है जो विघटन के लिए मौजूदा तरीकों पर प्रस्तावित विधि की प्रभावशीलता को दर्शाता है। आम तौर पर, टेलर श्रृंखला आधारित विधियों का उपयोग निरंतर समय प्रणालियों के सटीक विघटन के लिए किया जाता है लेकिन वे लीनियर टाइम वेरिंग (एलटीवी) सिस्टम के मामले में अप्रभावी पाए गए हैं। सटीक विघटन के लिए टेलर आधारित तरीकों के आलावा मौजूद एक वैकल्पिक विधि का सुझाव दिया गया है, और टेलर आधारित विधियों के उपयोग की असक्षमता का कारण बताया गया है | सटीक विघटन संस्करण प्राप्त करने के लिए यह समाकलन दृष्टिकोण आधारित विधि है। विभिन्न संख्यात्मक उदाहरणों के समर्थन के साथ सभी प्रस्तावित तकनीकों के लिए मेटलेब सिमुलेशन की मदद से पैरामीटर आधारित विश्लेषण किया गया है।

अध्याय 1 शोध प्रबंध का परिचय है जो प्रेरणा बताता है और बाद के अध्यायों के लिए पृष्ठभूमि बनाता है। अध्याय 2 में नॉनलीनियर क्रियाशील प्रणालियों पर जोर देने के साथ विघटन के क्षेत्र में महत्वपूर्ण कार्यों की विस्तृत समीक्षा करता है । अध्याय 3 निरंतर समय क्रियाशील प्रणालियों के विघटन का परिचय देते हुए , यह

लीनियर प्रणालियों के लिए उपयोग में लाये गए सटीक विघटन की विधियों का विस्तार नॉनलीनियर प्रणालियों तक करता है | अध्याय 4 ली अल्जेब्रिक ऑपरेटरों का उपयोग करके नॉनलीनियर प्रणाली को विघटित करने के लिए एक नया सिद्धांत प्रस्तावित करता है, जिसे अध्याय 5 में कुछ मानक नॉनलीनियर सिस्टम मॉडल्स के लिए प्रयोग में लाया गया है | अध्याय 6 एलटीवी सिस्टम के बारे के विघटन के लिए रणनीति का प्रस्ताव करता है। अंत में, समापन अध्याय में, शोध प्रबंध का सारांश प्रस्तुत किया गया है और अनुसंधान के भविष्य की दिशाओं की झलक प्रस्तुत की गई है।

Contents

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Abbreviations and Symbols

Abbreviation/ Symbol	Description
CTS	Continuous-time System
DTS	Discrete Time System
EDS	Exactly Discretizable System
ADC	Analog to Digital Converter
LTI	Linear Time Invariant
ODE	Ordinary Differential Equation
VPO	Van der Pol Oscillator
DO	Duffing Oscillator
FHN	FitzHugh Nagumo
HH	Hodgkin-Huxley
LTV	Linear Time Varying
NDS	Nonlinear Dynamical Systems
TLD	Taylor-Lie Discretization
NDE	Nonlinear Difference Equations
SDS	Sampled-Data System
T	Sampling time
$\mathbb{R}^n$	n Dimensional Euclidean space
$\mathbb{Z}^+$	Positive integers
μ	Control or damping parameter in VPO
δ	Damping factor in DO
α	Stiffness control parameter in DO
β	Amount of nonlinearity in DO
γ	Amplitude of periodic driving force in DO
ν	Membrane potential in FHN model
a, b	Constant parameters in FHN model
ω	Recovery variable in FHN model
I_{ext}	Stimulus current magnitude in FHN model
Φ	Discrete state transition matrix
Γ	Discrete input matrix
$D^r_{x_i}$	r^{th} Derivative of the first order differential equation function