

# BEHAVIOUR OF ANISOTROPIC GRANULAR MEDIA UNDER GENERAL STRESS SYSTEM

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BY  
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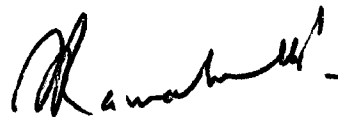
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C E R T I F I C A T E

This is to certify that the thesis entitled, "BEHAVIOUR OF ANISOTROPIC GRANULAR MEDIA UNDER GENERAL STRESS SYSTEM", being submitted by Mr. B.Shankariah to the Indian Institute of Technology, Delhi for the award of the degree of DOCTOR OF PHILOSOPHY is a record of the bonafide research work carried out by him. Mr. B.Shankariah has worked under my guidance for the submission of this thesis which to my knowledge has reached the requisite standard.

The thesis or any part there of, has not been submitted to any other University or Institution for the award of any degree or diploma.



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## ABSTRACT

The structure of the most of the granular media is anisotropic. An attempt is made to investigate the influence of inherent structural anisotropy on strength and deformation properties of granular media by conducting tests in universal triaxial apparatus.

The investigation was carried out on two uniformly graded sands, namely, Ottawa sand consisting of nearly spherical and rounded grains and crushed stone which consists of flaky and angular particles. Specimens were prepared by freely depositing saturated sand under water and vibrating in a specimen mould at different inclinations. Specimens were prepared at three different porosities. All the specimens were consolidated isotropically. Drained shear tests were conducted in Universal Triaxial Apparatus under monotonically varying mean normal stress conditions. Tests were conducted under the stress conditions of axisymmetric compression, general compression, plane strain compression, general extension and axisymmetric extension. Tests under different stress paths were conducted at identical values of mean normal stress at failure.

It was observed that the strength and deformation properties of granular media are influenced by structural anisotropy. In compression the angle of shearing resistance is constant in the region  $0^\circ < \theta < 30^\circ$ , decreases in the region  $30^\circ < \theta < 60^\circ$  and then again remains constant in the region  $60^\circ < \theta < 90^\circ$ , in which  $\theta$  is the angle made by the direction of deposition with the vertical axis of the specimen. In extension the angle of shearing resistance is constant in the region  $0^\circ < \theta < 30^\circ$ , increases in the

region  $30^\circ < \theta < 60^\circ$  and then again remains constant in the region  $60^\circ < \theta < 90^\circ$ . The variation of strength due to anisotropy is maximum under axisymmetric conditions. As the deviation from axisymmetric conditions increases, the strength anisotropy decreases continuously practically vanishing under plane strain conditions. The intermediate principal stress seems to suppress the structural anisotropy of the material.

With increase in deposition angle, the axial strain at any major principal stress ratio increases in compression but decreases in extension. The lateral strain in the direction of plane of preferred orientation is higher than the other lateral strain. In compression dilation increases with increase in the value of deposition angle.

With increase in the value of intermediate principal stress, the angle of shearing resistance increases upto plane strain condition. With further increase in the value of intermediate principal stress, the angle of shearing resistance decreases continuously upto axisymmetric extension. The value of angle of shearing resistance may be lower or higher than or equal to that in axisymmetric compression depending upon the anisotropy of the material.

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