ASYMPTOTIC MODELING OF MAGNETOSTRICTIVE MATERIAL - BASED STRUCTURES WITH APPLICATION TO ENERGY HARVESTING DEVICE DESIGN

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SEPTEMBER 2023

Asymptotic Modeling of Magnetostrictive Material based Structures with Application to Energy Harvesting Device Design

by

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Submitted

in fulfilment of requirements for the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

SEPTEMBER 2023

Certificate

This is to certify that the thesis entitled, **'Asymptotic Modeling of Magnetostrictive Material based Structures with Application to Energy Harvesting Device Design'** submitted by **Mr. Sajan Kumar Wahi** to the **Indian Institute of Technology Delhi** for the award of the degree of **Doctor of Philosophy** embodies original research work done by him under my supervision. In my opinion, the thesis work meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute. The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Date: September 14, 2023

Place: New Delhi, India

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Acknowledgments

I want to start by sincerely thanking my supervisor for helping me through the challenging transition from an inexperienced student to an independent researcher. Dr. Santapuri is more of a mentor and friend to me than a supervisor. She constantly listened to all the issues, which made working with her pleasant for me. She not only provided me with her knowledgeable thoughts on my field of study, but she also gave me advice on how to write better so that I could present the concept and findings in research journals.

Additionally, I'd like to thank the members of my student research committee (SRC), Prof. B. Prof. Ratnamala Chatterjee, for their encouraging words and perceptive criticism when my Ph.D. work was being evaluated. This is my chance to express my gratitude to Prof. Sriram Hegde and Mr. V. S. Rawat for providing the greatest resources possible for students in the computational lab. A special thanks go out to the Ministry of Human Resource and Development, Government of India, for providing the funding necessary for me to complete my Ph.D. study at IIT Delhi.

My senior colleagues, Dr. Gaurav Wats, Dr. Mohit Gupta, Dr. Adnan Ahmad, Dr. Bishweshwar Babu, Dr. Gargi, Dr. Anurag Singh, Dr. Sagar Saroha, Dr. Raushan Singh, Dr. Sriram K, Dr. Hasan, and Dr. PMG Bashir, mentored me during the early, challenging stage of my Ph.D. life. My friends Dr. Vickey Nandal, Sandeep Yadav, Vipin Kakkar, Abhishek Dahiya, Satyendra, Dr. Dharmender Jangra, Dr. Prashant Mittal, Kuldeep Yadav, and Ayush Srivastava and I will always remember the chats we had over a cup of tea. and my junior colleagues, Dr. Yadwinder Singh Joshan, Awantika, Devansh, Vikram, Aaquib, Ayush Varshney, Nikesh, Mayank, Manik, Srijan, Rishabh, Saurabh and each member of MOAM lab. The MOAM and computation labs have now grown to be like an extended family to me.

No words can adequately explain how grateful I am to my parents, sisters, brothers-in-law, closest friend, and my wife Tanima, who gave me unwavering emotional support and inspired me to finish my work. I'm hoping that by helping them for the rest of my life, I may earn back the confidence they placed in me. I want to make them prouder by improving myself and giving back to society as a teacher.

Sajan Kumar Wahi

Abstract

The present work deals with computationally efficient nonlinear modeling of magnetostrictive materials and their applications to device design. Application of the framework is demonstrated towards the design of novel magnetostrictive material-based energy harvesting devices.

The first part of the dissertation deals with the development of a computationally efficient nonlinear constitutive model for magnetostrictive materials. High computational efficiency is achieved through the use of local linearization (about easy axes) and discrete energy-averaging techniques. The model is applied to iron-gallium alloys (Galfenol) and tested for different magnetic field orientations relative to the easy axes. It is observed that the model accurately predicts both sensing and actuation characteristics while reducing the computation time by a large factor ($>$ 1000 times) when compared to the nonlinear energy minimization models. Furthermore, the average error observed in magnetostriction versus magnetic field ($\lambda - H$) and magnetic flux density versus magnetic field ($B - H$) curves is less than 3.5% with the error increasing at magnetic field orientations farther from easy axes, particularly at large magnetic field values. The model is subsequently extended to incorporate hysteresis effects. Finally, the model is integrated with a finite element framework to predict the response of a Galfenol rod transducer system, and parametric studies are performed for different current and pre-stress conditions to optimize the device's performance.

An in-house code for the constitutive model is developed using MATLAB to design the transducer and energy harvester device. A magnetostrictive rod transducer is modeled in both sensor and actuator operating modes. The magneto-mechanical system of governing equations is solved using the finite element software COMSOL Multiphysics, and the constitutive model is fed into the software using the MATLAB live-link option. The existing nonlinear constitutive model (Armstrong (1997b)) and locally linearized constitutive model ($M - H$ and $\lambda - H$) for magnetostrictive materials are compared for magnetic fields applied along [100], [110], [210], [211], [310], and [311] directions in Galfenol (with an easy axis along <100>). The constitutive model is generalized to 3D so that it can be used as a computational tool to design complex devices.

The second part of the dissertation deals with the modeling and design of magnetostrictive material-based energy harvesting devices. The computational framework developed in this work is applied to an unimorph beam-type energy harvesting device and its electrical output is analyzed. The effect of design modifications on induced voltage output and power output is investigated. The simulation results of a unimorph magnetostrictive beam-type energy harvester are validated by comparing them with existing literature (Deng and Dapino (2017b)). Subsequently, a tapered beam design and its voltage output are compared for four different taper designs with the simple unimorph design. An increase of $\approx$ 15% is observed in the output voltage for the best taper design. The design is further improved by a closed flux path designed using low-strength permanent magnets and a soft iron core. This modification resulted in an increase of 103 % in the voltage output at the same operating frequency and magnetostrictive material volume. A detailed analysis of this design is performed through a combination of simulation and experiments.

सारांश

प्रस्तुत शोध कार्य का सम्बन्ध चुंबकीय विरूपण सामग्री के अभिकल्नीय कुशल अरेखिक प्रतिरूपण और उपकरण अनुप्रयोग की रुपरेखा से है। ढांचे का अनुप्रयोग नवीन चुंबकीय विरूपण सामग्री आधारित ऊर्जा संचयन के उपकरण के लिए किया गया है।

शोध प्रबंध के प्रथम भाग का सम्बन्ध चुंबकीय विरूपण सामग्री के अभिकल्नीय कुशल अरेखिक प्रतिरूपण के विकास से संबधित है। उच्च अभिकल्नीय दक्षता को प्राप्त करने हेतु स्थानीय रेखीककरण (आसान अक्षों की ओर) और असतत ऊर्जा औसत तकनीक के उपयोग के माध्यम से प्राप्त किया जाता है। प्रतिरूपण को लौह-गैलियम मिश्र धातु (गैल्फेनॉल) और आसान अक्षों के सापेक्ष विभिन्न चुंबकीय क्षेत्र अभिविन्यासों के लिए परीक्षण किया गया। अवलोकन में यह पाया गया कि नवीन प्रतिरूपण अरेखिक ऊर्जा न्यूनीकरण की तुलना में, संवेदन और सक्रियण विशेषताओं दोनों की सटीक भविष्यवाणी करता है| इसके साथ-साथ नया प्रतिरूपण गणना समय (> 1000 गुना) को कम भी करता है। इसके अतिरिक्त, वक्र रेखीय चुंबकीय विरूपण बनाम चुंबकीय क्षेत्र (λ - H) और चुंबकीय प्रवाह घनत्व बनाम चुंबकीय क्षेत्र (B - H) में औसत 3.5 प्रतिशत की त्रुटि है, जिसका अधिकतम मान चुंबकीय क्षेत्र के आसान अक्षों से अधिक दूर होने पर और विशेषकर बड़े चुंबकीय क्षेत्र मानों पर दिखाई देता है। बाद में प्रतिरूपण को शैथिल्य प्रभावों को शामिल करने के लिए भी परिष्कृत किया गया है। अंत में, नवीन आरेखीये प्रतिरूपण सीमित तत्व ढांचे के साथ एकीकृत करके, गैल्फेनॉल छड़ ट्रांसड्यूसर प्रणाली की प्रतिक्रिया की भविष्यवाणी हेतु उपयोग में लाया गया। इसके साथ-साथ प्रतिरूपण का इस्तेमाल गैल्फेनॉल छड़ की विभिन्न विद्युत प्रवाह एवं विभिन्न पूर्व-तनाव स्थितियों के लिए प्राचलिक अध्ययन और उपकरण के प्रदर्शन को अनुकूलित बनाने में सहायक हैं।

MATLAB का उपयोग करते हुए संवैधानिक प्रतिरूपण के लिए एक आंतरिक संहिता विकसित की गई है, जिसका उपयोग ट्रांसड्यूसर और ऊर्जा संचयन उपकरण बनाने हेतु किया गया है। चुंबकीय विरूपण छड़ ट्रांसड्यूसर संवेदन और सक्रियण संचालन दोनों रूपों में प्रतिरूपित है। चुंबकीय यांत्रिक प्रणाली को चलाने वाली समीकरणों को सीमित तत्व प्रणाली COMSOL Multiphysics से हल किया जाता हैं और MATLAB Live-Link विकल्प का उपयोग करके संवैधानिक प्रतिरूपण को तंत्रांश में भरण किया जाता है। मौजूदा अरेखीय (आर्मस्ट्रांग (1997बी)) और स्थानीय रूप से रेखीकृत संवैधानिक प्रतिरूपण (M−H) और (λ−H) के बीच तुलना की गई है तथा दोनों का चुंबकीय क्षेत्र की विभिन्न दिशाओं जैसे [100], [110], [210], [211], [310] और [311] में तुलनात्मक प्रयोग दर्शाया गया है। संवैधानिक प्रतिरूपण को तीनों दिशाओं

में सामान्यीकृत किया गया है ताकि निर्मित अभिकल्नीय औजार का उपयोग जटिल उपकरण निर्माण की रूपरेखा में किया जा सके।

शोध प्रबंध का दूसरा भाग चुंबकीय विरूपण सामग्री के प्रतिरूपण और इन्हीं पर आधारित ऊर्जा संचयण करने वाले उपकरण से संबधित है। विकसित अभिकलनि ढांचे का इस्तेमाल unimorph beam प्रकार के ऊर्जा संचयन उपकरण और उसके विद्युत उत्पादन विश्लेषण के लिए किया गया है। विद्युत् उत्पादन तथा प्रेरित विभवांतर उत्पादन पर रूपरेखा संशोधनों के प्रभाव का गहन अध्यन किया गया। एक unimorph beam के ऊर्जा संचयण के परिणाम को उपलब्ध साहित्य (डेंग और डैपिनो (2017बी)) के साथ तुलना करके वैधता की पुष्टि की गई है। इसके बाद, एक सामान्य आयताकार beam की प्रतिरूपण और उसके विभवांतर उत्पादन की चार अलग-अलग शुन्डाकार प्रारूपो से तुलना की गई। सर्वोत्तम शुण्डाकार रूपरेखा के विभवांतर उत्पादन में ≈15 प्रतिशत की वृद्धि देखी गई। बंद चुंबकीय प्रवाह पथ का निर्माण, कम शक्ति वाले स्थायी चुम्बकों और नरम लौह अन्तर्भाग का उपयोग करके नवीन रूप-रेखा के माध्यम से सुधार किया गया। इस संशोधन के परिणामस्वरूप समान क्रिया संचालन आवृत्ति और चुंबकीय विरूपण सामग्री पर विभवांतर उत्पादन में 103 प्रतिशत की वृद्धि पाई गई। अनुकरण और प्रयोगों के संयोजन के माध्यम से विस्तृत विश्लेषण करके इस प्रारूप को प्रदर्शित किया गया है।

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μ Lagrange multiplier, see equation (3.1), page 32

$\mathcal{L}$ Lagrange equation, see equation (3.1), page 32

ϵ Dielectric constant, see equation (2.11), page 25

γ^σ Scaling parameter/Fit constant, see equation (2.30), page 30

$\gamma_1, \gamma_2, \gamma_3$ Direction cosine of general direction, see equation (3.24), page 41

λ Bulk magnetostriction, see equation (1.2), page 5

λ_{100}, λ_{111} Magnetostriction constants in $< 100 >$ and $< 111 >$ direction respectively, see equation (2.30), page 30

μ Magnetic permeability of material, see equation (2.11), page 25

μ_{nm}^σ Magnetic permeability nm component, see equation (1.1), page 3

Ω Smoothing constant, see equation (1.2), page 5

ϕ Azimuthal angle of spherical coordinates, see equation (1.2), page 5

ϕ Scalar potential, see equation (2.14), page 26

ρ Density of the structure, see equation (2.1), page 23

σ Electrical conductivity, see equation (2.11), page 25

θ Polar angle of spherical coordinates, see equation (1.2), page 5

ξ_{an}^k Anhysteretic domain volume fraction of the kth domain, see equation (3.17), page 39

ξ^{hyst} Volume fraction when hysteresis consider, see equation (3.27), page 42

μ_0 Vacuum permeability, see equation (2.10), page 25

$\overset{1}{c}_1, \overset{1}{c}_2, \overset{1}{c}_3$ Mutually independent easy axes directions, see equation (3.6), page 36

∂V_o Surface area integral, see equation (2.1), page 23

Φ Magnetic flux, see equation (2.23), page 28

ρ_e Electric free charge density, see equation (2.7), page 24

$\mathbf{A}$ Magnetic vector potential, see equation (2.12), page 25

$\mathbf{B}$ Magnetic flux density, see equation (2.8), page 24

$\mathbf{b}$ Body force acting on arbitrary control volume V_o, see equation (2.1), page 23

$\mathbf{D}$ Electric flux density, see equation (2.7), page 24

$\mathbf{E}$ Electric field vector, see equation (2.9), page 25

$\mathbf{H}$ Magnetic field vector, see equation (2.10), page 25

$\mathbf{J}$ Current density, see equation (2.10), page 25

$\mathbf{J}_e$ Eddy current density, see equation (2.15), page 26

$\mathbf{J}_s$ Applied current density, see equation (2.15), page 26

$\mathbf{T}$ Cauchy stress second order tensor, see equation (2.2), page 23

$\mathbf{t}$ Traction force, see equation (2.1), page 23

$\mathbf{W}^A$ Weighting function for magnetic potential equation, see equation (2.18), page 27

$\mathbf{W}^u$ Weighting function for displacement equation, see equation (2.4), page 23

ξ Damping ratio, see equation (4.4), page 68

B_Φ Φ component of magnetic flux density, see equation (2.24), page 28

B_n Magnetic flux density n^{th} component, see equation (1.1), page 3

d_{nij} Piezomagnetic coefficient nij component, see equation (1.1), page 3

d_{tip} Tip deflection of the beam, see equation (4.4), page 68

f_r The system fundamental first eigen frequency, see equation (4.4), page 68

H_n Magnetic field n^{th} component, see equation (1.1), page 3

K_0, K_1, K_2 Scaling parameter, Cubic anisotropy constant, Cubic anisotropy constant, see equation (2.28), page 29

k_p Material constant quantifying the number of pinning sites per unit volume, see equation (3.27), page 42

K_{norm} Normalization factor calculated for each applied magnetic field and stress, see equation (1.2), page 5

M Bulk magnetization, see equation (1.2), page 5

m^k Magnetic moment of a kth domain, see equation (1.2), page 5

m_1, m_2, m_3 Denote the direction cosines of magnetic moment vector along [100], [010], and [001] crystal directions, respectively, see equation (2.25), page 28

M_s Magnetic saturation, see equation (2.29), page 30

N Number of turns in the coil, see equation (2.24), page 28

s_{ijkl} Elastic compliance matrix ijkl component, see equation (1.1), page 3

S_{ij} Strain ij^{th} component, see equation (1.1), page 3

u Displacement vector, see equation (2.1), page 23

U_{total} Sum of energy: magneto-crystalline $U_{anisotropy}$, magnetoelastic $U_{magnetoelastic}$ and Zeeman energy U_{zeeman}, see equation (1.2), page 5

V Induced e.m.f, see equation (2.23), page 28

V_o Control volume, see equation (2.1), page 23