

**THE KINETIC TEARING INSTABILITY, AND THE OSCILLATING
TWO-STREAM INSTABILITY DRIVEN BY LOWER
HYBRID WAVES IN TOKAMAK PLASMAS**

by

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CERTIFICATE

This is to certify that the thesis entitled, "The Kinetic Tearing Instability, And The Oscillating Two Stream Instability Driven By Lower Hybrid Waves In Tokamak Plasmas" being submitted by Mr. P.V.Siva Rama Prasad for the award of the degree of 'Doctor of Philosophy', to the Indian Institute of Technology, Delhi, is a record of bonafied research work he has carried out under my guidance and supervision.

In my opinion, this thesis has reached the standard fulfilling the requirements of all regulations relating to the degree. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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PREFACE

The confinement and auxiliary heating of the plasma in a tokamak have been recognized as two most important fundamental problems in achieving the controlled thermonuclear fusion. The equilibrium plasma system is achieved by applying a toroidal and poloidal magnetic fields. The toroidal magnetic field is produced primarily from currents flowing in the toroidal field coils. The poloidal magnetic field is produced primarily by a toroidal current flowing in the plasma. The combination of these magnetic fields forms topologically two distinct magnetic surfaces: closed magnetic surfaces and open magnetic surfaces. The open surfaces intercept the inner wall of the reactor chamber. The plasma is contained within the closed surfaces. The boundary of the plasma must be kept away from the inner wall of the chamber, so that the impurity concentration can be minimum; these impurities are in general the consequence of collisions of runaway electrons with the inner wall. Otherwise, these impurities can greatly increase the radiation losses, and can create serious problem to the plasma heating to the thermo-nuclear temperatures. A limiter made of a refractory material such as molybdenum or tungsten is generally used to stop down the aperture of the plasma and protect the inner wall from runaway electrons.

A part of the thesis is devoted to the resistive tearing modes, and remaining part to the plasma heating by the externally launched lower hybrid waves.

The Resistive Tearing Modes

The equilibrium plasma system under the ideal magnetohydrodynamic equations of motion can support various types of eigen modes. The stability of the system with respect to these ideal MHD modes is fairly well understood in the cases of large-aspect-ratio, low- β and circular cross-section tokamaks, and it appears that the system is in stable equilibrium^{1,2,3} under appropriate conditions. But the cases of small-aspect-ratio, shaped cross-section, and high- β tokamaks are not well understood, and are the topics of current research. The equilibrium system can be stable with respect to the kink modes derived from the ideal MHD equations, if the toroidal and poloidal magnetic fields satisfy the Kruskal-Shafranov condition⁴. However, if the system is perturbed according to the ideal MHD equations, the perturbation becomes singular at a mode rational surface, S , at which $\vec{q} \cdot \vec{B}_0 = 0$, unless the perturbed field component normal to the surface at S vanishes, where \vec{q} is the wave vector of the mode and \vec{B}_0 the equilibrium magnetic field. Around this mode rational surface the ideal MHD equations are not valid. The resistivity term, introduced by

the electron-ion collisions and inertial effects, in the equation of magnetic field diffusion becomes significant, and is comparable to the convective term within a narrow annular region around the mode rational surface. As a consequence, the magnetic field within the annular region decouples from the plasma, and the mode can tear and reconnect the magnetic lines of force to form magnetic islands. However, away from the mode rational surface the plasma dynamics satisfies the ideal MHD conditions and therefore the magnetic lines of force are 'frozen-in' the plasma as in the internal kink modes. These modes corresponding to different values of wave vector, \vec{q} , are called as the resistive kink tearing modes^{5,6}, which are simply a generalization of internal kink modes. These modes grow at the cost of free energy stored in the equilibrium magnetic field within the annular region. In this process the system comes down to a lower magnetic energy state, and the rate at which the energy releasing depends upon the dynamics of the plasma around these mode rational surfaces. These modes have been extensively observed in tokamak discharges^{7,8,9}. It is believed that these modes are likely candidates to explain several phenomena limiting tokamak confinement, including plasma disruption^{10,11} and enhanced electron thermal transport^{12,13}.

It is obvious that any physical effect that influences

the plasma dynamics around the singular region can considerably modify the evolution of the tearing modes. In this thesis, the tearing modes driven by the electron inertia and background random fluctuations in the lower hybrid frequency range is studied. It may be believed that the fusion plasmas are relatively collision-free. The background random fluctuations can always exist in fusion plasmas from various microinstabilities. They can also arise from external excitations as in the case of radio frequency heating experiments. These short-wavelength lower hybrid fluctuations may be believed to modify significantly the plasma dynamics around the mode rational surfaces. This nonlinear interaction may result in either enhancement or stabilization of linear tearing modes driven by the electron inertia, or inducing new modes by the fluctuations. This calculation is presented, in detail, in chapter II.

Plasma Heating by Externally Launched Lower Hybrid Waves

At high temperatures, ohmic heating fails because the collision frequency decreases with the increase of temperature. Many auxiliary heating schemes like the radio frequency (rf) heating and neutral beam injection technique etc., have been introduced for the plasma heating to create required pressure, so that ions can overcome coulomb repulsive force to take part in nuclear fusion. For many

years, rf heating near the lower hybrid frequency has attracted much interest as a means of auxiliary heating. The availability of megawatt sources of microwave power in the lower hybrid range of frequencies and the use of waveguides external to the vacuum vessel to couple wave energy to the plasma makes this an extremely convenient method of plasma heating, especially from a technological stand point. The remaining part of the thesis is devoted to the plasma heating in tokamaks by the externally launched lower hybrid waves. Two of the most fundamental aspects of any lower hybrid heating experiment are accessibility¹⁴ and energy deposition, both parameterized by $n_{||} = (k_{||} C / \omega)$, where $k_{||} = (\vec{k} \cdot \vec{B}_0 / |B_0|)$ is the wave number parallel to the magnetic field, C the speed of the light and ω the wave frequency. By proper choice of $k_{||}$ and ω , one can obtain preferential heating of either ions or electrons.

In the lower hybrid frequency range by including the electromagnetic effects the dispersion relation yields two roots of k_{\perp}^2 , each corresponding to a different mode of wave propagation in the plasma, where k_{\perp} is the wave number perpendicular to the equilibrium magnetic field. One of these roots is the slow mode corresponding to the lower hybrid wave, and the second root is the fast mode. The lower hybrid wave, launched from a waveguide in the vacuum region surrounding the plasma, tunnels through the

evanescent region ($k_{\perp}^2 < 0$) at the plasma edge. The lower hybrid wave with $n_{\parallel} < n_c$ would not propagate into the plasma interior, but rather mode convert into the fast wave, where n_c is a critical value. In contrast to this, the lower hybrid wave with $n_{\parallel} > n_c$ would propagate into the plasma interior. Since $k_{\perp} \rightarrow \infty$ at the lower hybrid resonance layer, it is important to consider thermal effects in this region. In the vicinity of lower hybrid resonance layer, the wave propagation is essentially electrostatic in nature since $n_{\perp} \gg 1$, where $n_{\perp} = (k_{\perp} c / \omega)$. In the thermal plasma, the launched lower hybrid wave can not reach the resonance point, but instead would mode convert into an ion plasma wave^{15,16}. The ion plasma wave is a very short-wavelength mode, and is therefore strongly absorbed by linear ion Landau damping (assuming unmagnetized ions).

However, there has been experimental evidences¹⁷⁻²⁰ and theoretical predictions²¹ that nonlinear processes, i.e. parametric instabilities can become effective in absorbing the launched wave energy^{22,23} even before the wave reaches the ion plasma mode conversion layer. A variety of decay channels has been investigated²⁴⁻²⁷ in a homogeneous plasma with constant magnetic field. However, it is not straight forward to extrapolate these results for a realistic fusion-type plasma. The model has been improved by taking into account finite extent of the pump²⁸⁻³⁰ as well as density

and magnetic field inhomogeneities^{27,30}. It is well known that, in an inhomogeneous plasma, the parametric decay waves bend, thus allowing their energy to escape from the region of strong coupling^{31,32}. This phenomenon is called convective damping, since it reduces the growth rates and increases thresholds of these instabilities like a damping process.

In this thesis, Oscillating Two-Stream Instability (OTSI)³³⁻³⁷ of lower hybrid waves with finite wavelength is studied, considering thermal effects to the side-bands. Both experimental²⁰ and theoretical²⁴ work has shown the effectiveness of OTSI as a heating mechanism in the lower hybrid frequency range. The coupled mode equations, which describe lower hybrid side-bands under OTSI conditions, are in the form of higher-order coupled differential equations in ϕ_1 and ϕ_2 , where ϕ_1 and ϕ_2 are the electrostatic potentials of side-bands. These coupled equations are very complicated to solve analytically by explicitly considering the combined effect of density inhomogeneity, magnetic shear and finite extent of the pump on the side-bands. However, in chapter III, these coupled equations are solved for the growth rate and convective threshold of the instability by explicitly considering the combined effect of density inhomogeneity and magnetic shear, and assuming the pump wave as infinitely extended. The results so obtained are

compared to the ones obtained by considering the finite extent of the pump alone, in chapter IV. In chapter V, OTSI of lower hybrid waves in a cylindrical plasma column with a parabolic density profile and a constant axial magnetic field is studied; this calculation is more likely applicable to the beam-plasma systems.

A chapter-wise summary of the thesis is as follows.

CHAPTER - I: Introduction on the Resistive Tearing Modes

In this chapter, the physical interpretation of the tearing instability is presented. A brief review of previous work, followed by the significance of present work on the evolution of the tearing modes, is summarized.

CHAPTER-II: The Kinetic Tearing Modes Driven by the Electron Inertia and Background Lower Hybrid Fluctuations.

In this chapter, the modified tearing mode equations in $A_{\tau_{||}}$ and ϕ_{τ} , which describe the plasma dynamics around the mode rational surface, are obtained by employing the drift kinetic equation, considering the electron inertia and background random fluctuations in the lower hybrid frequency range, where $A_{\tau_{||}}$ and ϕ_{τ} are, respectively, the vector and scalar potentials associated with the tearing mode. These

equations are solved by employing the variational principle for the Fourier transformed perturbed current density. The dispersion relation so obtained is solved for the $m=1$ and $m \geq 2$ modes in some important limits.

The results are briefly summarized. The previous known $m=1$ and ≥ 2 tearing modes driven by the electron inertia are recovered from the present calculation. However, at a moderate level of fluctuations, $(|E_k|^2 / 8\pi n_0^0 T_e) \sim 10^{-5}$, new modes are found, where $|E_k|$ is the electric field associated with the lower hybrid modes with wave number \vec{k} , n_0^0 the equilibrium plasma density and T_e the electron temperature in energy units. The nonlinear interaction of $m=1$ inertial driven mode with the fluctuations induces a new mode. The growth rate and frequency of this new mode are relatively larger than that of linear $m=1$ mode. The current channel width is considerably increased by the fluctuations.

The inertial driven $m=2$ mode, in the limit $\omega^{1/2} \ll \beta_1$, would be replaced by a new mode driven by the nonlinear interaction with the fluctuations, where $\omega^{1/2}$ and β_1 are the current channel width and ion Larmor radius, respectively. The growth rate of this new mode is substantially larger than the linear $m=2$ mode. The current channel width is greatly increased by the fluctuations, and the frequency and growth rate of this new mode are comparable to each other.

However, the inertial driven $m=2$ mode, in the limit $|\omega|^{1/2} \gg \beta_i$ still survives even in the presence of the fluctuations. The nonlinear interaction with the fluctuations enhances mildly the current channel width as well as the growth rate of this mode.

CHAPTER-III: The Combined Effect of Density Inhomogeneity and Magnetic Shear on OTSI of Lower Hybrid Waves

In this chapter, the coupled mode equations, which describe the lower hybrid waves under OTSI conditions, are solved by considering explicitly the combined effect of density inhomogeneity and magnetic shear. The pump wave is assumed as infinitely extended. The coupled equations are converted into an uncoupled schroedinger equation in Fourier space by taking various transformations. This eigen value equation is solved by employing the WKB technique. It is found that OTSI is an absolute instability, the growth rate being an eigen value. Thermal effects introduce a condition, which limits the wave number spectrum, for existence of the instability. Expressions for the growth rate and convective threshold are obtained from the present calculation. It is found that this instability is likely to occur close to the ion plasma mode conversion layer (See Appendix B). These results are

applied to the parameters of Princeton Large Torus (PLT). The convective threshold turns out to be of the order of 3-7 KW/cm².

CHAPTER-IV: The Effect of Finite Extent of the Pump on OTSI of Lower Hybrid Waves

In this chapter, the coupled equations are solved, considering the finite extent of the pump and assuming the plasma as homogeneous with constant magnetic field. These equations are converted into an uncoupled second-order differential equation, and is solved for solutions both inside and outside the pump region. The solutions so obtained are matched at the boundaries to obtain a nonlinear dispersion relation, and an expression for the convective threshold is derived from the dispersion relation. To the parameters of PLT, the convective threshold turns out to be 10 KW/cm², which is relatively larger than the one introduced in the previous chapter. As the plasma density increases, the convective threshold decreases. This is because as the plasma density increases, the group velocities of the decay waves escaping across the width of the pump region decrease, and hence the convection losses decrease.

CHAPTER-V: OTIS of Lower Hybrid Waves in a Cylindrical Plasma Column

In this chapter, OTSI of lower hybrid waves in a cylindrical plasma column with a parabolic density profile and a constant axial magnetic field is studied, ignoring thermal effects to the side-bands. This calculation is more likely applicable to the beam-plasma systems. The pump wave equation in a cylindrical coordinate system is in the form of Laguerre equation. Linear parts of coupled mode equations are in the form of associated Laguerre equation. The coupled equations are therefore solved by employing the perturbation technique. An expression for the growth rate is obtained from this calculation.

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