# FIRST ORDER REACTANTS AND STATISTICAL THEORY OF TURBULENCE

by

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Submitted to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics.

1973

## CERTIFICATE

This is to certify that the thesis entitled 'First Order Reactants and Statistical Theory of Turbulence' which is being submitted by Mr. Sukaran Ram Patel for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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## ACKNOWLEDGEMENTS

It is with sincere appreciation that I acknowledge a debt of gratitude to many who have given me counsel and encouragement. My chief debt is to Dr. Prem Kumar, Assistant Professor, Department of Mathematics, Indian Institute of Technology, Delhi, for his invaluable advice and constructive criticism, without which this study could have never been completed.

I am grateful to Professor M.P. Singh, Visiting Professor, University of Cambridge, Cambridge, and to Professor M.K. Jain, Senior Professor, Department of Mathematics, Indian Institute of Technology, Delhi, for their interest in my progress.

I am particularly indebted to Dr. Robert G. Deissler, Senior Fluid Physicist of Lewis Research Laboratory, Cleveland, Ohio, U.S.A., to Dr. R. Dash, Engineering Department, University of Cambridge and to Dr. Yash Paul, Department of Mathematics, Indian Institute of Technology, Delhi, for their help and valuable suggestions during the preparation of this dissertation.

The early encouragement of Professor B.R. Bhonsle, Head of Department of Mathematics, Government Engineering College Jabalpur, Jabalpur, and of Dr. P.K. Bhattacharyya, Assistant Professor, Department of Mathematics, Indian Institute of Technology, Delhi, is gratefully acknowledged.

This work was supported by the Council of Scientific and Industrial Research, India, for which I am extremely grateful.

Thanks are due to all my colleagues and friends for their cooperation and help, particularly to Mr. S.C. Jethi, Defence Research and Development Laboratories, Hyderabad, and to Mr. R.G. Gupta of Computer Centre, Indian Institute of Technology, Delhi, for their help in numerical computations.

To Miss Neelam Dhody go my thanks for the immaculate care with which she has prepared the final typescript.

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# S Y N O P S I S

This thesis incorporating some aspects of the statistical features of turbulence contains six chapters. The introductory first chapter deals, in brief, with the present-day scope of turbulence, the gradual development of some related ideas and outstanding contributions of some of the pioneers in the field.

In the second chapter, the concentration fluctuations of a dilute contaminant undergoing a first order chemical reaction is discussed. Two-, three-and four-point correlation equations at a time have been obtained and to make the set of these equations determinate, the terms containing quintuple correlations are neglected in comparison to those containing lower order correlation terms. The solution obtained gives the state of concentration fluctuations in a homogeneous turbulence between its initial and final periods. If the contaminant is replaced by the temperature then the result for $C = 0$ (where C is the first order reaction rate constant) gives the decay law for temperature fluctuations in homogeneous turbulence which agrees with those found by other workers like Deissler and Loeffler and Corrsin.

In third chapter, the multi-point, multi-time correlation equations for the concentration correlation of a dilute contaminant undergoing a first order chemical reaction have been obtained for two-point, two-time correlation equations, the tripple correlations are neglected and the solution for the space time concentration correlations for the final period of decay has been obtained. The analysis is extended to the earlier times by considering three-point and three-time correlation equations and solving them after neglecting the quadruple correlations. The decay law for the concentration fluctuation is obtained.

In fourth chapter, we have retained the quadruple correlations of the third chapter and obtained the solution by neglecting the quintuple correlations. This chapter is different from chapter two in the sense that here, multi-point, multi-time correlations equations are considered whereas in that multi-point for a single time correlation equations are considered.

The fifth chapter deals with the problem of turbulent mass transfer and concentration fluctuations in a first order chemical reaction field with uniform velocity and concentration gradients. The analysis is

based on the equation of motion and a mass transfer of a dilute contaminant undergoing first order chemical reaction, wherefrom equations involving correlations between fluctuating quantities at two-points in the fluid have been derived. Although uniform mean velocity and concentration gradients are present in the fluid, the turbulence decays with time. In obtaining the solution the equations are converted into the spectral forms by taking their fourier transforms. Here it is assumed that the turbulence is sufficiently weak so that the tripple correlations are neglected as compared to the double correlations.

In chapter six, Pao's continuous spectral cascading concept for transfering turbulent energy and passive scalar quantities at large wave numbers is applied to study the transfer of turbulent magnetic energy at large wave numbers.

## LIST OF PAPERS

1. First Order Reactant in Homogeneous Turbulence Before the Final Period of Decay.
   To Appear in "Physics Fluids".
   This paper comprises the Second Chapter of the Thesis.

2. On First Order Reactants in Homogeneous Turbulence
   Communicated to the "International Journal of Engineering Science".
   It is the Third Chapter of the Thesis.

3. Multi-Point, Multi-Time Concentration Correlations and Decay of Homogeneous Turbulence,
   International Journal of Engineering Science. (To Appear)
   It comprises the Fourth Chapter of the Thesis.

4. Turbulent Mass Transfer and Concentration Fluctuation in a First Order Chemical Reaction Field with Uniform elocity and Concentration Gradients.
   Communicated to the "Applied Scientific Research".
   This is the Fifth Chapter of the Thesis.

5. Transfer of Turbulent Magnetic Energy at Large Wavenumbers.
   Communicated to the "International Journal of Engineering Science".
   This is the Sixth Chapter of the Thesis.

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