

**BLOCK CODED MODULATION USING TWO-LEVEL
GROUP CODES OVER DIHEDRAL AND
DICYCLIC GROUPS**

by

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Department of Electrical Engineering

submitted

in fulfilment of the requirements of the Degree of
DOCTOR OF PHILOSOPHY

to the



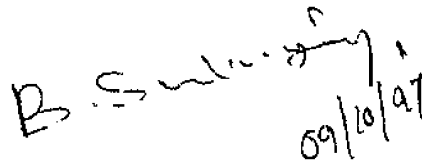
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*(Dedicated to the Sacred memory
of my respected Late Pitaji (Nanaji)
who was an apostle of truth*

CERTIFICATE

This is to certify that the thesis entitled "**BCM using Two-Level Group Codes over Dihedral and Oicyclic Groups**" being submitted by **Jyoti Bali** to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a record of the bonafide research work carried out by her under my supervision and guidance. The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.


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a.l.
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ABSTRACT

Two-Level block coded phase modulation schemes with a linear binary code C_1 and a linear code C_2 over Z_M , (the residue class integer ring modulo M) as component codes and a PSK signal (symmetric and asymmetric) set matched to a dihedral group D_M or a 4-dimensional signal set, called $(PSK)^2$ signal set, matched to a dicyclic group DC_M as basic signal sets have been studied. A pair of codewords one each from C_1 and C_2 , specify an n -tuple over D_M (or DC_M) and the set of all possible such n -tuples over D_M (or DC_M) is called a label code which, in general, is a subset, not necessarily a subgroup, of D_M (or DC_M). Extending the matched labelling component wise on all the elements of the label code results in a signal set in $2n$ dimensions (or in $4n$ dimensions) called the Two-Level signal space code where n is the length of the codes C_1 and C_2 .

For a group G , a subgroup of G^n under componentwise operation is a group code over G . A set of necessary and sufficient conditions on the component codes for the label code to result in a group code over D_M and DC_M are proved. For D_M the algebraic conditions obtained by Garelo and Benedetto for semidirect product group code are shown to be equivalent to part of the conditions obtained.

The performance of the signal space codes depend both on set of component codes and the matched labelling used. Given a component code pair different matched labellings will give different performance. We call the problem of identifying the best labelling the Initial Labelling Problem and provide a solution for a rather restrictive class of component codes for codes over D_M and show that Initial Labelling Problem does not arise for codes over DC_M , i.e., the performance of the signal space code depends only on the set of component codes and not on the matched labeling used. For codes over D_M , through a series of theorems it is shown that based on the ratio of the Hamming distances of the component codes several Euclidean

distance properties can be obtained for both symmetric and asymmetric PSK signal sets. Based on these results superiority of certain labellings over others is established. Moreover, conditions under which introduction of asymmetry to the symmetric PSK signal set will improve performance of the resulting signal space code are discussed and it is shown that when the conditions are satisfied upto certain angle performance improvement is guaranteed. These results are discussed in detail for the special cases of 4, 6, 8, 12 and 16-PSK signal sets and several classes of codes are identified and their coding gains tabulated.

It turns out that the conditions on the component codes to result in a group code over DC_M include the conditions for the same component codes to result in a group code over D_M . This means that such pairs of component codes can be used to label both the signal sets matched to D_M as well as DC_M and obtain different signal space codes. For such component codes we identify conditions, specifically rates of the code, under which one will perform better than the other.

Rotational invariance properties of the label codes, for symmetric and asymmetric PSK signal sets as well as $(PSK)^2$ signal sets are discussed and conditions for several angles of phase rotational invariance are proved. Based on these a two-stage differential encoding and decoding scheme is proposed. It is pointed out that the codes investigated in the thesis admit minimal trellises and hence are amenable to efficient soft decision decoding. Thesis concludes with a description of several possible directions for further research.

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AEDE Automorphic Euclidean Distance Equivalent

d_p MSED of a Two-Level signal space code with a group code over D_M as the label code

d_{DC} MSED of a Two-Level signal space code with a group code over DC_M as the label code

G_n Coding gain of a Two-Level signal space code with a group code over DC_M as the label code

G_{DC} Coding gain of a Two-Level signal space code with a group code over DC_M as the label code