BLOCK CODED MODULATION USING TWO-LEVEL GROUP CODES OVER DIHEDRAL AND DICYCLIC GROUPS

by

JYOTI BALI

Department of Electrical Engineering

submitted
in fulfilment of the requirements of the Degree of
DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI
HAUZ KHAS, NEW DELHI - 110016, INDIA

October, 1997

(Dedicated to the Sacred memory of my respected Late Pitaji (Nanaji) who was an apostle of truth

CERTIFICATE

This is to certify that the thesis entitled **"BCM using Two-Level Group Codes over Dihedral and Oicyclic Groups"** being submitted by **Jyoti Bali** to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree *of* **Doctor of Philosophy,** is a record of the bonafide research work carried out by her under my supervision and guidance. The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

B. Sundar Rajan
09/10/97

(B.Sundar Rajan)
Associate Professor
Department of Electrical Engineering
Indian Institute of Technology, Delhi
New Delhi 110 016
INDIA

ACKNOWLEDGEMENTS

I am highly indebted to Dr.B.Sundar Rajan for his continuing help, support and advise. His enthusiasm and open outlook always kept me going in my research. Occasionally, I was intimidated by his brilliance and hard working nature which I tried my best to imitate. I had my share of hardships to face but he would always help me overcome those. I sincerely thank him for that. The discussions I had with Dr.Adnan A. Zain at the early stages were very useful starting points. I would also like to thank Dr.S.D.Joshi, Prof.S.Prasad and Dr.Subiman Kundu for patiently listening to my proposal and providing useful suggestions. The discussions I had with Dr.Phillipe Piret and Dr.Oscar Y.Takeshita were invaluable for the work reported in this thesis.

I thank the Indian Institute of Technology, Delhi for providing me full-scholarship to pursue my research from July 1993 to Dec. 1995 and the Council of Scientific and Industrial Research (CSIR) for awarding me Senior Research Fellowship (SRF) from Jan.96 onwards.

I am grateful to God that I had colleagues like K.Chandrasekhar, Nidhi, Mona, Venkat, Atanu, Y.V.Joshi, Y.N.Singh, Brejesh, R.S.Tare, R.Rajesh Kumar, Monika, Anubala, Rekha Shoma and Nandini who provided support and friendship during these years.

I would like to take this opportunity to thank Prof.B.Kumar, Prof.A.B.Sharma and Dr.T.O'Farrell for developing my interest in research. I wish to express my gratitude to Prof.S.C.Dutta Roy, Prof.R.K.Patney, Prof.J.K.Chatterjee, Prof.G.S.Visweswaran, Prof.S.S.Jamuar and Dr.Ranjit Singh for their encouragement.

I would not have been able to sustain the pressures but for the love and affection of my sister Preeti and more recently my Brother-in-law Neeraj. Last but not the least, I thank my parents for pushing me, loving me and putting me back on track especially at a time when I had given up all the hopes of completing this work.

Above all, I thank God for giving me the strength to face all the odds that came my way and emerge as a winner.

Jyoti Bali

ABSTRACT

Two-Level block coded phase modulation schemes with a linear binary code C_1 and a linear code C_2 over Z_M, (the residue class integer ring modulo *M)* as component codes and a PSK signal (symmetric and asymmetric) set matched to a dihedral group D_M or a 4-dimensional signal set, called $(PSK)^2$ signal set, matched to a dicyclic group DC_M as basic signal sets have been studied. A pair of codewords one each from C_1 and C_2, specify an n -tuple over D_M (or DC_M) and the set of all possible such n -tuples over D_M (or DC_M) is called a label code which, in general, is a subset, not necessarily a subgroup, of D_M (or DC_M). Extending the matched labelling component wise on all the elements of the label code results in a signal set in $2n$ dimensions (or in An dimensions) called the Two-Level signal space code where n is the length of the codes C_1 and C_2.

For a group G, a subgroup of G'' under componentwise operation is a group code over *G. A* set of necessary and sufficient conditions on the component codes for the label code to result in a group code over D''_M and DC''_M are proved. For D''_M the algebraic conditions obtained by Garello and Benedetto for semidirect product group code are shown to be equivalent to part of the conditions obtained.

The performance of the signal space codes depend both on set of component codes and the matched labelling used. Given a component code pair different matched labellings will give different performance. We call the problem of identifying the best labelling the Initial Labelling Problem and provide a solution for a rather restrictive class of component codes for codes over D_M and show that Initial Labelling Problem does not arise for codes over DC_M, i.e., the performance of the signal space code depends only on the set of component codes and not on the matched labeling used. For codes over D_M, through a series of theorems it is shown that based on the ratio of the Hamming distances of the component codes several Euclidean

distance properties can be obtained for both symmetric and asymmetric PSK signal sets. Based on these results superiority of certain labellings over others is established. Moreover, conditions under which introduction of asymmetry to the symmetric PSK signal set will improve performance of the resulting signal space code are discussed and it is shown that when the conditions are satisfied upto certain angle performance improvement is guaranteed. These results are discussed in detail for the special cases of 4, 6, 8, 12 and 16-PSK signal sets and several classes of codes are identified and their coding gains tabulated.

It turns out that the conditions on the component codes to result in a group code over DC_M include the conditions for the same component codes to result in a group code over D_M. This means that such pairs of component codes can be used to label both the signal sets matched to D_M as well as DC_M and obtain different signal space codes. For such component codes we identify conditions, specifically rates of the code, under which one will perform better than the other.

Rotational invariance properties of the label codes, for symmetric and asymmetric PSK signal sets as well as $(PSK)^2$ signal sets are discussed and conditions for several angles of phase rotational invariance are proved. Based on these a two-stage differential encoding and decoding scheme is proposed. It is pointed out that the codes investigated in the thesis admit minimal trellises and hence are amenable to efficient soft decision decoding. Thesis concludes with a description of several possible directions for further research.

CONTENTS

		Page No.
	List of Principal Symbols and Abbreviations	*viii*
	List of Tables	*x*
	List of Figures	*xi*
Chapter 1	Introduction	
	1.1 Background & Preliminaries	1
	1.2 Motivation & Scope of the thesis	12
	1.3 Outline of chapters	16
Chapter 2	Two-Level Group codes over D_M and DC_M	
	2.1 Introduction	18
	2.2 Signal sets matched to D_M and DC_M	18
	2.2.1 Groups D_M and DC_M	19
	2.2.2 Signal sets matched to D_M	21
	2.2.3 Signal sets matched to DC_M	24
	2.3 Group codes over D_M and DC_M	30
	2.3.1 Two-Level group codes over D_M	31
	2.3.1.1 Semidirect product group codes	34
	2.3.2 Characterization for Two-Level group codes over DC_M	38
	2.4 Discussion	41
Chapter 3	Two-Level Signal space codes: Initial Labelling Problem, Lower Bounds on MSED, Coding Gain	
	3.1 Introduction	43
	3.2 Euclidean distance Equivalent Labellings	44
	3.3 Initial Labelling Problem	49
	3.3.1 Initial Labelling Problem for D_M	49
	3.3.2 Initial Labelling Problem for DC_M	50
	3.4 Bounds on the MSED of $\mu^n(C)$	51
	3.4.1 A lower bound on the MSED of $\mu^n(C)$ over D_M	53
	3.4.1.1 An elaborate bound	55
	3.4.2 A lower bound on the MSED of $\mu^n(C)$ over DC_M	57
	3.5 Expressions for coding gain	58
	3.6 Discussion	60

Chapter 4 **Euclidean distance properties of** Two-Level **SPSK signal** space **codes**

4.1 Introduction 64
4.2 Case I: d_s<d, 67
4.3 Case II: J$_($> J, 73
4.4 Special Case: 4-SPSK 77
4.5 Special Case: 6-SPSK 79
4.5.1 Class I: C, is the ternary parity check code and 'C, is the binary repetition code 82
4.5.2 Class II: C_r is the ternary Hamming code and C_x is the binary repetition code 83
4.5.3 Class III: $C_r = Z_3''$ and C, is any (n,k,d_x) binary linear code 84
4.5.4 Class IV: $C, = Z_3''$ and C, is the binary parity check code 85
4.5.5 C_r is a good ternary code with known d, and C, is the binary repetition code 86
4.5.6 Comparison of 6-SPSK codes 86

4.6 Special Case: **8-SPSK** 91
4.7 More Special Cases: 12-SPSK and 16-SPSK 96
4.7.1 Special case: 12-SPSK 96
4.7.2 Special Case: 16-SPSK 98
4.8 Discussion 98

Chapter 5 **Euclidean distance properties of Two-Level APSK signal** space **codes**

5.1 Introduction 100
5.2 Case I: d_s <d_r 102
5.3 Case II: d_s >d_r 109
5.4 Special case: 4-APSK 111
5.5 Discussion 113

Chapter 6 Signal space codes over $(PSK)^2$ signal set

6.1 Introduction 115

6.2 The function $[R]$ 117

6.3 Comparison of coding gains of signal space codes over D_{2M} and DC_M 121

6.4 Discussion 133

Chapter 7 Rotational Invariance & Decoding

7.1 Introduction 134

7.2 Rotational invariance of SPSK signal space codes 135

7.3 Rotational invariance of APSK signal space codes 136

7.4 Rotational invariance of PSK^2 signal space codes 138

7.5 Differential encoding & decoding 141

7.6 Minimal Trellises 149

7.7 Discussion 154

Chapter 8 Conclusion

8.1 Summary of results 156

8.2 Scope for further research 157

References 159

Appendix A 164

Appendix B 166

Appendix C 168

Bio-Data of the author 172

AEDE Automorphic Euclidean Distance Equivalent

d_p MSED of a Two-Level signal space code with a group code over D_M as the label code

d_{DC} MSED of a Two-Level signal space code with a group code over DC_M as the label code

G_n Coding gain of a Two-Level signal space code with a group code over DC_M as the label code

G_{DC} Coding gain of a Two-Level signal space code with a group code over DC_M as the label code