

COMPUTATIONALLY EFFICIENT FORMULATIONS FOR MODEL PREDICTIVE CONTROL AND THEIR APPLICATIONS

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COMPUTATIONALLY EFFICIENT FORMULATIONS FOR MODEL PREDICTIVE CONTROL AND THEIR APPLICATIONS

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CERTIFICATE

This is to certify that the thesis entitled **Computationally Efficient Formulations for Model Predictive Control and their Applications** submitted by **Keerthi Chacko** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

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A handwritten signature in black ink, reading "Keerthi", written in a cursive style with a long horizontal stroke extending to the left.

Keerthi Chacko

ABSTRACT

Model Predictive Control (MPC) is one of the most widely used control techniques in the industry. The popularity of MPC is apparent as it handles many input-output variables and constraints simultaneously. Despite the advantages, MPC faces some challenges. The computation time taken for MPC is extortionate as it has to solve an optimization problem at each sampling instant. In the quest to reduce computational effort, researchers have proposed various techniques like move blocking, sub-optimal control, etc. These strategies suffer from stability and performance guarantee issues. With a focus on extending the application domains of MPC, this thesis proposes several computationally efficient formulations for Model Predictive Control. This thesis attempts to address the computational issue from various angles: varying prediction horizon, reduction of constraints, event triggering framework, and approximate dynamic programming approach. To reduce the number of variables in the optimization problem, the thesis proposes a simple technique by which a short prediction horizon can be determined. The analysis of the disturbance effect in the proposed method led to a superior algorithm compared to the initial proposal. In the practical implementation of MPC, the constraints are active for only a fraction of the entire plant operation time. Hence, we propose the constraint-aware formulation in the optimization problem. In this technique, the constraints are determined that may be active in the immediate future and considered in the optimization problem. The thesis further presents an event-triggering formulation that reduces average computation time. When the system response is in line with the desired response, the control sequence doesn't have to be recalculated but uses previously calculated control inputs. The design of suitable triggering conditions enabled it to handle disturbances effectively. In addition, the thesis also presents an approximate dynamic programming formulation to suitably approximate cost function. Without much loss of performance, the computational time is reduced significantly. A Multi-tank system with nonlinear system dynamics is used to carry out the validation of the proposed algorithms.

Keywords: Model Predictive Control, Varying Horizon, Active Constraints Aware, Event triggering, Intermittent Feedback Control, Approximate Dynamic Programming, Nonlinear Systems, Tracking Control, Multi Tank System.

सार

मॉडल प्रेडिक्टिव कंट्रोल (एमपीसी) उद्योग में सबसे व्यापक रूप से इस्तेमाल की जाने वाली नियंत्रण तकनीकों में से एक है। एमपीसी की लोकप्रियता स्पष्ट है क्योंकि यह एक साथ कई इनपुट-आउटपुट चर और बाधाओं को संभालती है। फायदे के बावजूद, एमपीसी को कुछ चुनौतियों का सामना करना पड़ता है। एमपीसी के लिए लिया गया गणना समय जबरन वसूली है क्योंकि इसे प्रत्येक नमूना क्षण में एक अनुकूलन समस्या को हल करना है। कम्प्यूटेशनल प्रयास को कम करने की खोज में, शोधकर्ताओं ने विभिन्न तकनीकों जैसे मूव ब्लॉकिंग, उप-इष्टतम नियंत्रण आदि का प्रस्ताव दिया है। ये रणनीतियाँ स्थिरता और प्रदर्शन गारंटी के मुद्दों से ग्रस्त हैं। एमपीसी के आवेदन डोमेन का विस्तार करने पर ध्यान केंद्रित करने के साथ, यह थीसिस मॉडल प्रेडिक्टिव कंट्रोल के लिए कई कम्प्यूटेशनल रूप से कुशल फॉर्मूलेशन का प्रस्ताव करती है। यह थीसिस विभिन्न कोणों से कम्प्यूटेशनल मुद्दे को संबोधित करने का प्रयास करती है: अलग-अलग पूर्वानुमान क्षितिज, बाधाओं में कमी, घटना ट्रिगरिंग फ्रेमवर्क और अनुमानित गतिशील प्रोग्रामिंग दृष्टिकोण। अनुकूलन समस्या में चर की संख्या को कम करने के लिए, थीसिस एक सरल तकनीक का प्रस्ताव करती है जिसके द्वारा एक लघु पूर्वानुमान क्षितिज निर्धारित किया जा सकता है। प्रस्तावित विधि में गड़बड़ी प्रभाव के विश्लेषण ने प्रारंभिक प्रस्ताव की तुलना में एक बेहतर एल्गोरिथम का नेतृत्व किया। एमपीसी के व्यावहारिक कार्यान्वयन में, बाधाएं पूरे संयंत्र संचालन समय के केवल एक अंश के लिए सक्रिय हैं। इसलिए, हम अनुकूलन समस्या में बाधा-जागरूक सूत्रीकरण का प्रस्ताव करते हैं। इस तकनीक में, बाधाओं को निर्धारित किया जाता है जो तत्काल भविष्य में सक्रिय हो सकते हैं और अनुकूलन समस्या में विचार किया जा सकता है। थीसिस आगे एक घटना-ट्रिगरिंग सूत्रीकरण प्रस्तुत करता है जो औसत गणना समय को कम करता है। जब सिस्टम प्रतिक्रिया वांछित प्रतिक्रिया के अनुरूप होती है, तो नियंत्रण अनुक्रम को फिर से गणना करने की आवश्यकता नहीं होती है, लेकिन पहले से गणना किए गए नियंत्रण इनपुट का उपयोग करता है। उपयुक्त ट्रिगरिंग स्थितियों के डिजाइन ने इसे गड़बड़ी को प्रभावी ढंग से संभालने में सक्षम बनाया। इसके अलावा, थीसिस उपयुक्त रूप से अनुमानित लागत फंक्शन के लिए एक अनुमानित गतिशील प्रोग्रामिंग फॉर्मूलेशन भी प्रस्तुत करता है। प्रदर्शन के बहुत नुकसान के बिना, कम्प्यूटेशनल समय काफी कम हो जाता है। प्रस्तावित एल्गोरिदम के सत्यापन को पूरा करने के लिए नॉनलाइनर सिस्टम डायनामिक्स के साथ एक मल्टी-टैंक सिस्टम का उपयोग किया जाता है।

कीवर्ड: मॉडल प्रेडिक्टिव कंट्रोल, अलग-अलग क्षितिज, सक्रिय बाधाएं जागरूक, इवेंट ट्रिगरिंग, आंतरायिक फीडबैक कंट्रोल, अनुमानित डायनामिक प्रोग्रामिंग, नॉनलाइनर सिस्टम, ट्रैकिंग कंट्रोल, मल्टी टैंक सिस्टम।

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List of Abbreviations

ADP: Approximate Dynamic Programming

CA-MPC: Constraint Aware- Model Predictive Control

CA-MW-MPC: Constraint Aware- Monotonically Weighted Nonlinear Model Predictive Control

CDF: Cumulative Distribution Function

CE-NMPC: Computationally Efficient- Nonlinear Model Predictive Control

CR-MPC: Constraint Reduction Model Predictive Control

DP: Dynamic Programming

ETC: Event Triggering Control

ET-MPC: Event Triggering- Model Predictive Control

LTI: Linear Time-Invariant

MIMO: Multi Input Multi Output

MPC: Model Predictive Control

NMPC: Nonlinear Model Predictive Control

RHC: Receding Horizon Control

SAS: Switched Affine System

List of Symbols

s	scalar s
\mathbb{N}	Set of natural numbers
\mathbb{Z}	Set of integers
\mathbb{R}	Set of real numbers
\mathbb{Z}^+	Set of nonnegative integers
\mathbb{R}^+	Set of nonnegative real numbers
\mathbb{R}^n	n -dimensional Euclidean space
$\mathbb{R}^{m \times n}$	space of $m \times n$ real matrices
\exists	there exists
\forall	for all
x_k	State at $k - th$ instant
\mathcal{X}	Set of all permissible states
\mathcal{U}	Set of all permissible control
J_k^*	Optimal cost at $k - th$ instant
\tilde{J}_{k+1}	Candidate cost at $k + 1 - th$ instant
$P \succ 0$	Positive definite matrix P
$P \succeq 0$	Positive semidefinite matrix P
$P \prec 0$	Negative definite matrix P
$P \preceq 0$	Negative semidefinite matrix P
$\lambda_{\min}(P)$	minimum eigenvalue of the matrix P
$\lambda_{\max}(P)$	maximum eigenvalue of the matrix P
$\ \mathbf{s} \ $	Euclidean norm of the vector \mathbf{s}
\mathbf{I}_n	Identity matrix with dimension $n \times n$