TWO STAGE OPTIMIZATION IN TRANSPORTATION AND ASSIGNMENT PROBLEMS

by

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Certificate

This is to certify that the thesis entitled "Two-Stage Optimization in Transportation and Assignment Problems" which is being submitted by Sonia for the award of the degree of Doctor of Philosophy in Mathematics to the Indian Institute of Technology, Delhi, is a bona fide record of research work carried out by the author under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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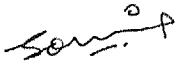

(Sonia)

Abstract

In the thesis entitled **"Two-Stage Optimization in Transportation and Assignment Problems"** the thrust is on the development and conceptualization of a special class of time minimizing transportation and assignment problems called Two Stage Transportation/Assignment Problem. This work is primarily an attempt to develop the polynomial time algorithms for two stage optimization in time minimizing transportation and assignment problems in different situations defined by relevant constraints. These are variants of two-dimensional time minimizing transportation and assignment problems.

Quite often hierarchy in decision making process and/or preferred status given to some sources or source-destination links, facilities or jobs leads to two stage optimization, in particular, and multistage optimization over transportation or assignment polytope in general.

The proposed two stage optimization problems are 'concave minimization problems', in which the decision variables are partitioned into two disjoint categories: Stage-I (or Level-I) and Stage-II (or Level-II) decision variables. Objective functions to be minimized in both the stages are concave functions. A master problem is associated with a two stage optimization problem. This master problem also involves minimizing a concave objective function. Various feasible solutions of two-stage optimization problem are generated by solving this master problem and its restricted versions.

The thesis comprises of five chapters. The first introductory chapter pertains to the review of the work related to the proposed study and contains the summary of the research work presented in this thesis.

Chapter 2 is focussed on the study of two different imbalanced bottleneck trans-

portation problems wherein certain restrictions like storage capacity limitations or lack of maintenance facilities at the production floor insist on the shipment of goods in two stages. In the first problem the total availability of a homogeneous product at the sources exceeds the minimum requirement of the same at the destination points while in the second problem the same is known to lie in a given interval. A feasible solution yielding the sum of the transportation times in the two stages is aimed.

In Chapter 3 minimization of the sum of the two concave functions over transportation polytopes in standard time minimizing transportation problems is studied. The two concave functions involved in both the problems pertain to the shipment times over two disjoint categories of source-destination links. The two categories of links are optimally generated by the optimal solution in one problem while these two categories are known in advance in the other problem. The objective is to determine the shipment schedules to minimize the sum of the two shipment times.

Optimal determination of primary facilities to first perform the given set of primary jobs followed by the optimal completion of the remaining (secondary) jobs is considered in a standard time minimizing assignment problem in Chapter 4. When the primary jobs are not known apriori, the optimal partition of the jobs into primary and secondary jobs is discussed next. The goal in both the problems is to minimize the sum of the makespan times in the primary and the secondary stages. Further this study on time minimizing assignment problem is extended to the case of time minimizing transportation problem.

In the last, in Chapter 5, a brief explanation is given as to how two stage optimization in transportation and assignment is some sort of a colouring problem in networks [18]. Unlike in concave minimization problems, the proposed solution methodologies in all the chapters are neither parametric nor branch-and-bound techniques. Very judicious enumeration of feasible solutions in each case is carried out. The proposed solution strategies make use of well studied conventional time

minimizing transportation problems, time minimizing assignment problems, cost minimizing transportation problems and cost minimizing assignment problems. Convergence of the algorithms to termination is accelerated by defining restricted versions of these problems in an appropriate manner. As these problems are solvable by polynomial bound algorithms, the proposed algorithms are also polynomial bound.

A possible extension of the two stage optimization to the bulk transportation problems, three dimension transportation problems, the three dimensional assignment problems and the networks is mentioned for a possible future work.

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