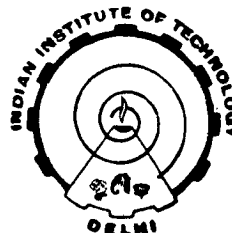


A MATHEMATICAL MODEL FOR SOIL EROSION SIMULATION

by

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*Thesis submitted to the
Indian Institute of Technology, Delhi
for the award of the degree of
DOCTOR OF PHILOSOPHY*

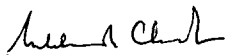


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December, 1988

CERTIFICATE

This is to certify that the thesis entitled " A Mathematical Model for Soil Erosion Simulation" being submitted by Manjit Singh Ahluwalia, to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy , is a record of bonafide research work carried out by him under our guidance and supervision. In our opinion, the thesis reached the requisite standard fulfilling the requirements of the regulation relating to the said degree. The material contained in this thesis has not been submitted, in part or full, to any other university or institute for the award of any degree or diploma.



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A handwritten signature in cursive script, appearing to read 'Manjit Singh Ahluwalia', written in black ink on a white background.

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SYNOPSIS

Introduction

Soil erosion due to wind, water and associated poor land management practices appears to be increasing day by day. Complete understanding of the soil erosion process and its reasonable estimation is important for better control of soil erosion and thereby maintaining maximum levels of agricultural production. A mathematical model reasonably representing the erosion processes based on physical laws suits to such requirements.

Mathematical Modeling of Soil Erosion Processes

Universal Soil Loss Equation (USLE) is widely used empirical relationship that uses rainfall energy as the main driving force for predicting erosion losses. It is not possible to predict soil loss on event basis using USLE. Also, it does not help in estimating the deposition within the catchment. Erosion losses can also be predicted using simulation models which utilize both the rainfall and runoff rates as the driving force for estimating the soil loss from the catchments.

Distributed parameter approach, utilizing the finite difference schemes for the solution of St. Venant equations, has been in use since 1970 for estimating the sediment yield from the catchments. Difficulties encountered in representing complex geometry and boundary conditions did not popularise the use of working with these models.

In order to improve the performance of soil erosion simulation models, the study uses finite element method to the constitutive

equations of flow and dynamic form of sediment continuity equation.

The sequence of the steps employed in the development of the continuous soil erosion simulation model presented herein is as follows:

1. Estimation of Infiltration (GAML model)
2. Estimation of evapotranspiration model (Reddy's model)
3. Soil water balance
4. Water routing (St. Venant equations, using finite element method)
5. Sediment routing (dynamic sediment continuity equation, using finite element method)
6. Verification of the model
 - (a) Comparison of the numerical solution of flow routing with analytical solution and the reported laboratory measured data
 - (b) Verification of the soil erosion simulation model using upland runoff and erosion data.

Estimation of infiltration:

The infiltration has been estimated by using Green and Ampt model (1911) modified by Mein and Larson (1973) for steady rainfall conditions and later on improved by Chu (1978) for unsteady conditions.

Green, W.H., and Ampt, G.A. 1911. Studies on soil physics. I. The flow of air and water through soils. Journal of Agricultural Science, Vol. 4, pp. 1-24.

Mein, R.G., and Larson, C.L. 1973. Modeling infiltration during a steady rain. Water Resources Research, Vol. 9. No. 2, pp. 384-394.

Chu, S.T. 1978. Infiltration during an unsteady rain. Water Resources Research, Vol. 14, No. 3, pp. 461-466.

Herein, this model is referred to as Green-Ampt-Mein-Larson (GAML) model. The GAML model uses the two-stage infiltration model because in reality the infiltration from rainfall normally occurs in two stages, before and with surface ponding. It conceptually considers a rainfall event to consist of a number of uniform intensity intervals. For each of the interval it identifies through the use of stage indicators, whether to use the pre-ponding or post-ponding equation for estimation of infiltration. The GAML model is a two parameter model. Its parameters can be easily measured by conducting experiments in the laboratory or in the field. The other input of the GAML model, saturation moisture deficit, is assumed to be equal to fillable porosity to simulate infiltration for the first rainfall event of the rainy season. The infiltration for the subsequent event has been estimated using soil water balance. Moreover, its potential to be used for unsteady rainfall conditions, through the use of stage indicators, has increased its utility for field problems.

Evapotranspiration (ET):

Reddy's model (Reddy, 1983) has been employed to estimate the daily evapotranspiration rate. The major inputs, climate, soil and crop parameters are easily measurable and it has been tested for different soils, climates and crop conditions. Advantage of this model is that it differentiates between fallow and cropped areas by a factor that remains constant for the fallow conditions but varies for the cropped areas during its growth process.

For arriving at the temporal distribution of ET, it is assumed that basal ET, at about 10 per cent of the daily ET, occurs throughout the day and night (Singh and Kumar, 1983). Ninety per cent of the daily ET, due to solar energy is assumed to occur during day time (over 12-hours period). Superposition of sinusoidal ET over the basal value results in 95 per cent of daily ET occurring during the day and only 5 per cent during 12-hours night.

Soil water balance:

Continuous models monitor the soil moisture status during the storm and between the storms. This has been achieved by input-output budgeting. The infiltration is the major input to the soil profile. Soil water in excess of field capacity of the soil is lost in the form of deep percolation. The available soil water is always subjected to evapotranspiration that continuously depletes the moisture of the soil profile.

Water routing:

Water routing for overland and channel phases of the catchment has been done using St. Venant equations. To increase the efficiency of the solution to the constitutive flow equations kinematic wave approximation is made which still allows the solution to be obtained for wide range of the flow conditions. The application of this approximation results in the uniform flow conditions and the flow has been estimated using Manning's equation.

Singh, S.R., and Kumar, A. 1983. Analysis of soil water dynamics: I. Modeling water uptake by plant roots. Vol. 20, No.3 and 4, pp. 79-100.

Finite element method (FEM) has been used to obtain the numerical solution of the constitutive equations of flow. Galerkin residual method in conjunction with both linear and cubic interpolation functions has been used for element matrix formulation. Explicit time integration technique is used in the numerical solution.

The flow characteristics obtained from the water routing are used in the solution of dynamic sediment continuity equation.

Sediment routing:

Sediment routing for both phases of the catchment has been performed by using dynamic form of the sediment continuity equation. The available sediment load at the end of the flow regime is compared with the sediment transport capacity of the flow. The limiting one of the two is actual sediment load. Sediment transport capacity has been computed using Yalin bed-load equation (1963). Its advantage is that it is well adapted for shallow flow conditions. Also, it has been modified by Foster (1972 b) to accommodate non-uniform particle sizes in sediment flow.

The sediment continuity uses sediment detachment rate that is estimated by summing up the inter-rill and rill detachment rates. Inter-rill erosion takes place because of raindrop impact. It has been assumed that the transport capacity of the flow in the inter-rill areas is just sufficient to supply the sediment

Yalin, Y.S. 1963. Expression for bed-load transportation. Journal of the Hydraulics Division, ASCE, Vol. 89, No. HY 3, pp. 221-250.

Foster, G.R. and Meyer, L.D 1972 b. Transport of soil particles by shallow flow. Transactions of the ASAE, Vol. 15, No.1, pp. 99-102

to the rills to be transported down the slope. If the transport capacity of the flow is more than the available sediment load due to inter-rill erosion, rill erosion occurs. In this study, equations as suggested by Foster (1982) have been used to estimate the inter-rill and rill erosion rates in the overland phase of the catchment that has to be routed down the slope. The sediment discharge from the overland flow phase becomes lateral inflow for the channel phase.

Finite element method is used to obtain the numerical solution of the sediment continuity equation. Galerkin residual method is used in conjunction with both linear and cubic interpolation functions for element matrix formulation.

Verification of the model:

Finite element solutions, using both linear and cubic interpolation functions, of the flow routing component of the model has been compared with the analytical solution (Eagleson, 1970) and the reported laboratory data (Crawford and Linsley, 1966). The FEM solutions have compared reasonably well with both the analytical solution and the reported laboratory data.

-
- Foster, G.R. 1982. Modeling the erosion process. In Hydrologic Modeling of small watersheds, ASAE Monograph No. 5 (Ed. Haan, CT, Johnson, D.L. and Brakensiek, D.L). American Society of Agricultural Engineers. Michigan. pp. 409-434.
- Eagleson, P.S. 1970, Dynamic Hydrology. Mc Graw-Hill. Book Co., New York.
- Crawford, N.H., and Linsley, R.K. 1966. Digital simulation in hydrology Stanford Watershed Model IV. T.R. 39. Stanford. California Dept. of Civil Engineering. Stanford University.

The soil erosion simulation model has been verified by comparing the simulated total runoff and soil loss amounts, resulting from a number of rainfall events, with the reported field data (Akan and Ezen, 1982). A similar comparison was made by using the ICRISAT data*. A reasonably good agreement has been found while comparing the simulated results with the published data for both cases.

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Akan, A.O. and Ezen, S.C. 1982. Mathematical simulation of erosion on graded terraces. In: Recent Developments in the Explanation and Prediction of Erosion and Sediment yield. Proceedings of the Exeter, U.K., Symposium. IAHS Publ, No. 137. (Ed. D.E. Walling) pp. 221-228.

* Data have been acquired through personal communication from:

1. Dr. R.C. Sachan (Agricultural Engineer)
2. Dr. Sardar Singh, (Soil Physicist)
ICRISAT, Patancheru P.O., A.P. 502324, INDIA.

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