EXACT 2D BUCKLING SOLUTION AND GEOMETRICALLY NONLINEAR ZIGZAG THEORY OF HYBRID PIEZOELECTRIC BEAMS

by

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Piezoelectric beams
Hybrid beams

Dedicated to

My Parents

Certificate

*This is to certify that the thesis entitled "**Exact 2D Buckling Solution and Geometrically Nonlinear Zigzag Theory of Hybrid Piezoelectric Beams**" being submitted by Mr. Md. Naushad Alam to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.*

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

A benchmark exact two-dimensional (2D) coupled piezothermoelasticity solution is presented for the buckling and free vibration analysis of simply-supported symmetric piezoelectric hybrid beams and infinite panels under initial inplane electrothermomechanical loads. The governing equations for the buckling/vibration mode are obtained using nonlinear Lagrangian strain-displacement relations and second Piola-Kirchhoff stress components. The effect of initial transverse normal strain, which is significant for thermoelectric loading, is accounted for. Both the open and closed circuit conditions are considered. Benchmark results for the critical values of axial strain, potential and temperature for buckling, and for the natural frequencies under initial stress are presented for hybrid highly inhomogeneous composite, cross-ply composite, angle-ply composite and soft-core sandwich laminates.

An efficient coupled geometrically nonlinear 1D zigzag theory is presented for hybrid beams under electrothermomechanical loading, by extending a recently developed linear theory. The geometric nonlinearity is included due to deflection in Von Karman sense. This theory considers a global third order variation with a layerwise linear variation across the thickness for the axial displacement, a sublayerwise linear variation for the electric potential and the thermal field, and a sublayerwise quadratic variation for the deflection, which directly accounts for the transverse extensibility due to the thermal and electric fields. The theory is as efficient as a smeared third order shear deformable theory (FSDT), since the displacement field is finally expressed in terms of only three displacement variables by satisfying exactly the conditions on the transverse shear stress at the layer interfaces and at the top and the bottom surfaces. The governing nonlinear coupled field equations of momentum and charge balance, and the variationally consistent boundary conditions are derived using the extended Hamilton's principle. The nonlinear theory is used to obtain the initial buckling and free vibration response of symmetrically laminated hybrid simply-supported beams under initial inplane electrothermomechanical loading.

A new finite element (FE) model of the 1D zigzag theory is developed for the linear static and dynamic analysis of hybrid beams under electrothermomechanical loading. The beam element has two nodes with four mechanical degrees of freedom and a variable number of electric

potential degrees of freedom for each node. The cubic Hermite interpolation is used for the deflection and electric potentials, and linear interpolation is used for the axial displacement and the shear rotation. The temperature is interpolated using quadratic Lagrange interpolation function introducing an additional node at the centre of the beam element. A consistent rectangular six-node thermal finite element with a quadratic interpolation along the axial direction and linear interpolation along the thickness direction is used for the thermal analysis. A coupled geometrically nonlinear consistent third order theory (CTOT) and its finite element model for the linear case are also developed. This theory satisfies the shear traction-free conditions at the top and the bottom surfaces of the beam for any electrical boundary conditions unlike the available third order theory (TOT).

The new zigzag theory is assessed for buckling and free vibration response under initial electrothermomechanical load, by comparing its results for simply-supported hybrid beams with the exact 2D piezothermoelasticity solutions developed herein. The zigzag theory is also assessed for the linear response of hybrid beams with various boundary conditions, by comparing the results of its 1D FE model with the 2D FE results obtained using plane stress element in ABAQUS. This assessment is presented for linear static and dynamic response under electromechanical loads and static response under thermal loads. The zigzag theory results are also compared with the TOT, the CTOT and the FSDT, which have the same efficiency as the zigzag theory. Such comparison with the 2D piezothermoelasticity solutions establishes the accuracy of the displacement field approximations made in the 1D theories. Hybrid beams with highly inhomogeneous test type, composite and soft-core sandwich laminates are considered for the assessment.

The major contributions of this work are a) development of an exact 2D coupled piezothermoelasticity solution for buckling and free vibration of simply supported symmetric hybrid beams under initial axial stress due to electrothermomechanical loading, b) development of an efficient geometrically nonlinear 1D zigzag theory for electrothermomechanical analysis of hybrid beams and its finite element model for the linear case, c) development of a coupled geometrically nonlinear consistent third order theory along with its finite element model for the linear case, and d) an assessment of the developed 1D theories for (1) buckling and free vibration response under initial stresses by comparison with the analytical exact 2D solution of simply-supported beams, and (2) for linear static and dynamic response of hybrid beams with other boundary conditions by comparison with the 2D finite element solutions. It is concluded that the zigzag theory is as efficient as the FSDT, TOT and CTOT, and yet yields generally far superior and accurate results for static, dynamic and buckling response of moderately thick to thin hybrid composite and sandwich beams for all kinds of lay-ups, loadings and boundary conditions.

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