

**ELECTROMAGNETOHYDRODYNAMIC FLOW THROUGH POROUS
MEDIA - MULTISCALE APPROACH**

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**DEPARTMENT OF MECHANICAL ENGINEERING
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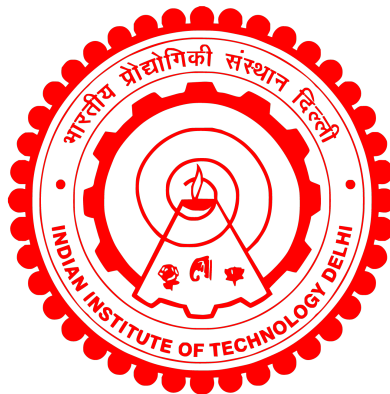
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Certificate

This is to certify that the thesis entitled “**ELECTROMAGNETOHYDRODYNAMIC FLOW THROUGH POROUS MEDIA - MULTISCALE APPROACH**”, submitted by **Ms. Promasree Majumdar** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy** in the Department of Mechanical Engineering, is a record of bona fide research work carried out under my guidance and supervision. The thesis has reached the standard fulfilling the requirements of the regulations relating to the Degree.

The work presented in this thesis is original and has not been submitted in part or in full for the award of any other degree or diploma of any other University or Institute.

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Abstract

Transport phenomena in porous media play a central role in diverse scientific and engineering applications in subsurface reservoirs, filtration and separation technologies, energy devices, and microfluidic platforms. While classical Darcy-scale models adequately describe pressure-driven single-phase flows, they do not account for the complex ways in which externally applied electric and magnetic fields alter momentum transport, modify flow pathways, and induce anisotropic behavior. Such field-driven mechanism, termed electromagnetohydrodynamic (EMHD) effects, are increasingly exploited in applications such as enhanced oil recovery, groundwater remediation, fuel cell design, and lab-on-a-chip actuation. This thesis addresses this critical knowledge gap by developing a rigorous multiscale framework for EMHD flow through porous media and establishing electro-magneto permeability, a field-dependent macroscopic property that quantifies how external fields modify hydraulic transport. The work systematically addresses fundamental gaps related to pore-scale EMHD mechanisms, their upscaling through multiscale homogenization, and the resulting macroscopic constitutive laws applicable to engineered and natural porous structures. The thesis considers both idealized model geometries and realistic porous structures, examining how microstructural parameters, anisotropy, tortuosity, porosity, and field orientation collectively influence EMHD-driven transport. Starting from the fundamental microscale governing equations, including the Navier-Stokes equations coupled with electrokinetic and Lorentz forcing, and ion transport laws, the thesis employs multiscale homogenization to systematically upscale pore-scale EMHD physics into generalized Darcy-scale constitutive relations. The work first examines single-phase EMHD flow in uncharged porous media, where electromagnetic fields act on conducting fluids without electric double layer (EDL) effects. The resulting homogenized equations reveal field-induced corrections. The derived permeability tensor reflects both microscale-geometry-induced and field-induced anisotropy, demonstrating that flow can be enhanced, suppressed, or redirected depending on field orientation and microstructure. The framework is then extended to charged porous media, where the presence of EDLs introduces electroosmotic forces and charge density variations in addition to Lorentz forces. A new generalized Darcy law is derived, and the corresponding permeability tensor explicitly incorporates these multiscale interactions. Building on these single-phase results, the thesis then investigates two-phase EMHD flow through charged porous media. The presence

of interfaces introduces additional complexities, including capillary pressure, saturation-dependent relative permeability, wettability, and interfacial charge. A generalized two-phase EMHD Darcy formulation is derived in which the classical relative permeabilities are augmented by saturation- and field-dependent EMHD terms. The final part of the thesis integrates the developed theoretical framework with real porous media, using paper substrates as a representative natural fibrous material. Micro-CT based reconstructions are used to simulate pore-scale EMHD transport and quantify permeability modifications. This thesis establishes a rigorous and unified multiscale approach for modeling EMHD flow in porous media. The introduction of the electro-magneto permeability both in tensor form and as a scalar equivalent measure offers a new paradigm for characterizing field-driven transport in porous media. The framework developed here lays a strong foundation for future studies on reactive EMHD flows, electro-magneto-chemo-mechanical coupling, and optimization of field-assisted transport in both engineered and natural porous materials.

सार

छिद्रयुक्त माध्यमों में परिवहन घटनाएँ (Transport phenomena) उपसतही भंडारों, निस्पंदन एवं पृथक्करण प्रौद्योगिकियों, ऊर्जा उपकरणों तथा माइक्रोफ्लूइडिक प्लेटफॉर्म जैसे विविध वैज्ञानिक एवं अभियान्तिक अनुप्रयोगों में एक केंद्रीय भूमिका निभाती हैं। यद्यपि पारंपरिक डार्सी-स्तरीय मॉडल दाब-प्रेरित एकल-चरण प्रवाह का पर्याप्त वर्णन करते हैं, वे उन जटिल प्रभावों को नहीं समाहित कर पाते जिनके माध्यम से बाह्य रूप से लगाए गए विद्युत एवं चुंबकीय क्षेत्र संवेग परिवहन को परिवर्तित करते हैं, प्रवाह पथों को संशोधित करते हैं तथा विषमदिशीय (anisotropic) व्यवहार उत्पन्न करते हैं। ऐसे क्षेत्र-प्रेरित तंत्र, जिन्हें इलेक्ट्रोमैग्नेटोहाइड्रोडायनामिक (EMHD) प्रभाव कहा जाता है, उन्नत तेल पुनर्प्राप्ति, भूजल शोधन, फ्यूल सेल डिज़ाइन तथा लैब-ऑन-ए-चिप एक्ट्यूएशन जैसे अनुप्रयोगों में तीव्रता से उपयोग में लाए जा रहे हैं। यह शोध-प्रबंध इस महत्वपूर्ण ज्ञान-अंतर को संबोधित करता है, जिसमें छिद्रयुक्त माध्यमों के माध्यम से EMHD प्रवाह के लिए एक सुदृढ़ बहुस्तरीय (multiscale) ढांचा विकसित किया गया है तथा इलेक्ट्रो-मैग्नेटो पारगम्यता (electro-magneto permeability) की स्थापना की गई है जो एक क्षेत्र-निर्भर स्थूल (macroscopic) गुण है और यह मात्रात्मक रूप से दर्शाता है कि बाह्य क्षेत्र हाइड्रॉलिक परिवहन को किस प्रकार संशोधित करते हैं। यह कार्य छिद्र-स्तर के EMHD तंत्रों, उनके बहुस्तरीय समानीकरण (homogenization) द्वारा अपस्केलिंग, तथा अभियान्तिक एवं प्राकृतिक छिद्रयुक्त संरचनाओं पर लागू होने वाले स्थूल संवैधानिक नियमों से संबंधित मूलभूत अंतरालों को क्रमबद्ध रूप से संबोधित करता है।

शोध-प्रबंध में आदर्शकृत मॉडल ज्यामितियों तथा यथार्थवादी छिद्रयुक्त संरचनाओं दोनों का अध्ययन किया गया है, जिनमें सूक्ष्म-संरचनात्मक मापदंडों, विषमदिशीयता, टॉर्चुएसिटी, छिद्रता तथा क्षेत्र की दिशा का EMHD-प्रेरित परिवहन पर संयुक्त प्रभाव विश्लेषित किया गया है। मूल सूक्ष्म-स्तरीय नियंत्रक समीकरणों-नैवियर-स्टोक्स समीकरण, इलेक्ट्रोकाइनेटिक एवं लॉरेंज़ बलों तथा आयन परिवहन नियमों के युग्मन सहित-से आरंभ करते हुए, इस शोध में बहुस्तरीय समानीकरण का उपयोग कर छिद्र-स्तर की EMHD भौतिकी को सामान्यीकृत डार्सी-स्तरीय संवैधानिक संबंधों में व्यवस्थित रूप से रूपांतरित किया गया है।

कार्य का पहला भाग अआवेशित छिद्रयुक्त माध्यमों में एकल-चरण EMHD प्रवाह की जाँच करता है, जहाँ विद्युत द्विस्तर (EDL) प्रभावों के बिना चालक द्रवों पर विद्युत-चुंबकीय क्षेत्र कार्य करते हैं। प्राप्त समानीकृत समीकरण क्षेत्र-जनित संशोधनों को उजागर करते हैं। निष्कर्षतः प्राप्त पारगम्यता टेन्सर सूक्ष्म-ज्यामिति-प्रेरित तथा क्षेत्र-प्रेरित दोनों प्रकार की विषमदिशीयता को दर्शाता है और यह स्थापित करता है कि प्रवाह को क्षेत्र की दिशा एवं सूक्ष्म संरचना के आधार पर बढ़ाया, घटाया या पुनर्निर्देशित किया जा

सकता है। इसके पश्चात यह ढांचा आवेशित छिद्रयुक्त माध्यमों तक विस्तारित किया गया है, जहाँ EDL की उपस्थिति इलेक्ट्रोऑस्मोटिक बलों और आवेश घनत्व के परिवर्तनों को लॉरेंज़ बलों के साथ जोड़ती है। एक नया सामान्यीकृत डार्सी नियम व्युत्पन्न किया गया है, जिसमें पारगम्यता टेन्सर स्पष्ट रूप से इन बहुस्तरीय अंतःक्रियाओं को समाहित करता है।

इन एकल-चरण परिणामों के आधार पर, शोध-प्रबंध का अगला भाग आवेशित छिद्रयुक्त माध्यमों में द्वि-चरण EMHD प्रवाह की पड़ताल करता है। इंटरफेस की उपस्थिति अतिरिक्त जटिलताएँ उत्पन्न करती है, जैसे केपिलरी दाब, संतृप्ति-आश्रित सापेक्ष पारगम्यता, गीलेपन (wettability) तथा अंतरापृष्ठीय आवेश। एक सामान्यीकृत द्वि-चरण EMHD डार्सी सूत्रीकरण व्युत्पन्न किया गया है, जिसमें पारंपरिक सापेक्ष पारगम्यताओं को संतृप्ति- एवं क्षेत्र-निर्भर EMHD पदों से समृद्ध किया गया है।

शोध-प्रबंध का अंतिम भाग विकसित सैद्धांतिक ढांचे को वास्तविक छिद्रयुक्त माध्यमों के साथ एकीकृत करता है, जहाँ कागज़ आधारित सबस्ट्रेट को एक प्रतिनिधि प्राकृतिक रेशेदार पदार्थ के रूप में अपनाया गया है। माइक्रो-सीटी आधारित पुनर्निर्माणों का उपयोग कर छिद्र-स्तर EMHD परिवहन का अनुकरण किया गया है तथा पारगम्यता में होने वाले संशोधनों का परिमाणीकरण किया गया है। समग्र रूप से, यह शोध-प्रबंध छिद्रयुक्त माध्यमों में EMHD प्रवाह के मॉडलन हेतु एक सुसंगत एवं कठोर बहुस्तरीय दृष्टिकोण स्थापित करता है। टेन्सर रूप तथा उसके समतुल्य स्केलर माप दोनों में इलेक्ट्रो-मैग्नेटो पारगम्यता की अवधारणा, छिद्रयुक्त माध्यमों में क्षेत्र-प्रेरित परिवहन के चरित्रांकन के लिए एक नया दृष्टिकोण प्रस्तुत करती है। यह ढांचा भविष्य में अभिक्रियात्मक EMHD प्रवाह, इलेक्ट्रो-मैग्नेटो-केमो-यांत्रिक युग्मन तथा अभियान्त्रिक एवं प्राकृतिक छिद्रयुक्त पदार्थों में क्षेत्र-सहायित परिवहन के अनुकूलन हेतु सशक्त आधार प्रदान करता है।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
List of Figures	xii
List of Tables	xvi
Symbols	xvii
1 Introduction	1
1.1 Overview	1
1.2 Flow in porous media: Classification, Parameters and Mathematical Modelling	4
1.2.1 Classification of Porous Media: Natural and Artificial	4
1.2.2 Flow regimes and Mathematical models	5
1.2.3 Challenges in Modelling	7
1.2.4 Descriptions of Flow: Pore-Scale vs. Continuum-Scale	7
1.2.5 Multiscale modeling	8
1.2.6 Important Parameters	9
1.3 Applications	10
2 Literature Review	12
2.1 Single-Phase Flow Through Porous Media	12
2.1.1 Introduction	12
2.1.2 Classical continuum theory	13
2.1.3 Pore-scale modelling (DNS, LBM, PNM)	14

2.1.4	Multiscale homogenization and upscaling	15
2.2	Multiphase Flow Through Porous Media	16
2.2.1	Classical continuum theory: Relative Permeability, Capillary Pressure	16
2.2.2	Pore-scale modelling: DNS, LBM, PNM	17
2.2.3	Multiscale upscaling and effective constitutive relations	18
2.3	Electromagnetohydrodynamic (EMHD) flow	20
2.4	Research Gaps	22
2.5	Research Objectives	23
2.6	Electromagneto-Permeability	24
2.7	Thesis Outline	25
3	Single Phase EMHD Flow in an uncharged Porous Media	27
3.1	Introduction	27
3.1.1	Applications of Single-Phase EMHD Flow in Uncharged Porous Media	28
3.2	Governing Equations	31
3.2.1	Current Density	31
3.2.2	Lorentz Force in the Fluid	32
3.2.3	Navier-Stokes Equation	32
3.3	Asymptotic homogenization	33
3.3.1	Scale separation	34
3.3.2	Multiscale Differential Operator	34
3.3.3	Scaling Analysis of the Momentum Equation	35
3.3.4	Dimensionless Numbers and Order of Magnitude Analysis	35
3.3.5	External electric field formulation	36
3.3.6	Magnetic field	40
3.3.7	Continuity Equation	43
3.3.8	Fluid flow	44
3.3.9	Solving the complete cell problem	46
3.3.10	Microscale solution	48
3.4	Behaviour of the Electromagneto-Permeability Tensor $\mathbf{k}_{em}$	49
3.4.1	Origin of Anisotropy	50

3.5	Results for 2D	52
3.5.1	Domain	52
3.5.2	Numerical Implementation	53
3.5.3	Model Validation	54
3.5.4	Mesh Convergence Study	54
3.5.5	Solution	55
3.5.6	Effect of Magnetic field on EM-Permeability	57
3.5.7	Effect of aspect ratio on EM-Permeability	58
3.5.8	Effect of Orientation	61
3.6	Conclusion	63
4	Single phase EMHD flow in a Charged Porous Media	66
4.1	Introduction	66
4.1.1	Charged Porous Media: Physical Origin and Characteristics	67
4.1.2	Electro-Magnetohydrodynamic (EMHD) Flow in Charged Media	68
4.1.3	Applications of EMHD Flow in Charged Porous Media	69
4.2	Governing Equations	72
4.2.1	Electrical Double Layer (EDL) Formulation	72
4.2.2	Fluid Flow	73
4.2.3	Scaling Analysis	74
4.2.4	Asymptotic homogenization	75
4.2.5	Solving the complete cell problem	77
4.2.6	Microscale solution	78
4.3	Behaviour of Electromagneto Permeability Tensor K_{em} in charged porous media	79
4.4	Results	81
4.4.1	Domain	81
4.4.2	Numerical Implementation	81
4.5	Conclusion	86
5	Two phase EMHD flow through charged porous media	88
5.1	Problem Formulation	88
5.1.1	Phase field formulation	89

5.1.2	Fluid Flow Formulation	93
5.2	Multiscale Framework and Asymptotic Homogenization	95
5.2.1	Electrical Double Layer (EDL) Equation	97
5.2.2	Electric Field	98
5.2.3	Magnetic field	99
5.2.4	Continuity equation	101
5.2.5	Cahn-Hilliard equation	102
5.2.6	Momentum equation	103
5.3	Derivation of Effective properties	105
5.3.1	Behaviour of Electromagneto-permeability tensor	107
5.4	Numerical Implementation	110
5.4.1	Properties	110
5.4.2	Effect of Magnetic field on EM-Permeability	112
5.4.3	Effect of wall potential on EM-Permeability	114
5.4.4	Effect of $\bar{\kappa}$ on EM-Permeability	116
5.4.5	Velocity Field Characteristics	118
5.4.6	Effect of saturation on Capillary Pressure	120
5.5	Conclusion	122
6	Real porous media: Paper Substrate	124
6.1	Introduction	124
6.2	Literature Review on Paper Microfluidics	125
6.2.1	Applications of Paper Microfluidics	126
6.2.2	Passive Flow Mechanisms in Paper Microfluidics	126
6.2.3	Active Flow Control Using External Fields	127
6.3	Reconstruction of REV	128
6.4	Results	129
6.4.1	Uncharged	129
6.4.2	Charged	132
6.5	Conclusion	135
7	Conclusion and Future Scope	137
7.1	Conclusions	137

7.2 Future Scope	139
Bibliography	140

List of Figures

1.1	Regular and irregular patterns of the 2-D porous structures by lithography: a) Pore body-throat network, b) regular arrays of pillars, c) cross hatched grid, d) irregular pattern representing the structure of a thin layer of real rock, and e) irregular inter-connected channels [1]. . . .	5
1.2	Numerical modelling strategies for flow in porous media.	7
1.3	Multiscale Modelling methodology	8
1.4	a) Isotropic and homogeneous; b) Anisotropic and Homogeneous; c) Isotropic and Heterogeneous; d) Anisotropic and Heterogeneous.	9
1.5	Applications: a) Combined Enhanced Oil Recovery (EOR) and CO_2/H_2 storage; b) Targeted Drug Delivery; c) Fuel Cell; d) Microfluidic setups .	11
3.1	A 3D representation of a porous media with external fields. Orange arrow indicating the direction of the resultant flow.	27
3.2	Key application areas of EMHD flow in uncharged porous media	29
3.3	A 3D periodic unit cell with spherical solid at the center.	32
3.4	Schematic of a two-dimensional representation of porous media (a) Macroscopic medium of length L (b) Periodic unit cell (Ω) of length l comprising of fluid phase Ω_f with an elliptical solid phase Ω_s at the centre where Γ is the boundary of unit cell and Γ_{fs} defines the solid-fluid interface.	53
3.5	Present numerical result compared with the results reported by [2]. . . .	54
3.6	Mesh convergence study of EM-permeability	55
3.7	Field distributions for $B^{(0)} = 1.25$ T, $\alpha = 2.6$, porosity $n = 0.9$, and orientation $\theta = 45^\circ$. (a) Velocity field $v^{(0)}$, (b) Voltage correction $V^{(1)}$, (c) Pressure correction $p^{(1)}$	56
3.8	Effect of magnetic field $B^{(0)}$ on EM-permeability K_{em} : (a) for the entire $B^{(0)}$ range and (b) up to the critical $B^{(0)}$ value.	57

3.9	Effect of aspect ratio (α) on electro-permeability (K_{em}) tensor for different porosity values (n). The other constants $\theta = 0^\circ$, $B^{(0)} = 1.25$ T	60
3.10	Effect of orientation angle (θ) on (a) $K_{em,xx}$ and $K_{em,yy}$ components of the EM-permeability tensor and (b) the scalar equivalent EM-permeability K_{em} . The other constants are $\alpha = 1.8$ and $n = 0.8$	63
3.11	Plots depicting velocity field for three different orientation angles $\theta = 0^\circ, \theta = 45^\circ$, and $\theta = 90^\circ$. The other constants are $\alpha = 1.8$, $\frac{\partial V^{(0)}}{\partial Y_1} = -10V$, $B^{(0)} = 1.2566$ T, and $n = 0.8$	64
3.12	Fig. Streamline plots for three different orientation angles $\theta = 0^\circ, \theta = 45^\circ$, and $\theta = 90^\circ$. The other constants are $\alpha = 1.8$ and $n = 0.8$	64
4.1	3D representation of a porous media with external fields. Orange arrows indication the two different forces acting.	66
4.2	Application areas of EMHD flow through charged porous media	70
4.3	a) Negatively charged porous media; b) Electrical Double Layer formation on the surface of the porous media shown in a periodic unit cell.	72
4.4	A 3D periodic unit cell with charged solid at the center.	74
4.5	Effect of Wall Zeta potential on K_{em}	82
4.6	Effect of electrolyte concentration on K_{em}	84
4.7	Effect of Magnetic field on K_{em}	86
5.1	A representation of a porous media of length L and periodic unit cell of length l on the right side comprising of fluid phase α with an spherical solid phase β at the centre where Γ is the boundary of unit cell, $\Gamma_{\alpha\beta}$ defines the solid-fluid interface and Γ_{wn} defines the interface between two fluid phases (wetting (w) and non-wetting (nw))	90
5.2	A 3D schematic of a periodic unit cell illustrating the forces acting on a negatively charged solid sphere representing the porous medium material. An externally applied electric field in the $+y$ direction and a magnetic field in the $+z$ direction generate a Lorentz force in the $+x$ direction. Additionally, the negatively charged surface interacts with the applied electric field, producing an electroosmotic (EO) force in the $+y$ direction.	107

5.3	(a) Velocity field $\bar{\mathbf{v}}^{(0)}$, (b) Voltage correction $\bar{V}^{(1)}$, (c) Phase field $\phi^{(0)}$, (d) Electric double layer potential $\bar{\Psi}^{(0)}$ for the case when $\bar{B}^{(0)} = 1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$ in $+y$ -direction, $\bar{\Psi}_w = -1$, $\bar{\kappa} = 0.1$	112
5.4	Effect of magnetic field $\bar{B}^{(0)}$ on (a) Total EM-permeability K_{em} , (b) relative non-wetting phase EM-permeability K_{rnw} , (c) relative wetting phase EM-permeability K_{rw} . Other constants are $\bar{\Psi}_w = -1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$, $\bar{\kappa} = 0.1$	113
5.5	Effect of wall zeta potential $\bar{\Psi}_w$ on total EM-permeability K_{em} . Other constants are $\bar{B}^{(0)} = 1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$, $\bar{\kappa} = 0.1$	114
5.6	Effect of wall zeta potential $\bar{\Psi}_w$ on (a) relative non-wetting phase EM-permeability K_{rnw} , (b) relative wetting phase EM-permeability K_{rw} . Other constants are $\bar{B}^{(0)} = 1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$, $\bar{\kappa} = 0.1$	115
5.7	Effect of $\bar{\kappa}$ on (a) total EM-permeability K_{em} , (b) relative non-wetting phase EM-permeability K_{rnw} , (c) relative wetting phase EM-permeability K_{rw} . Other constants are $\bar{B}^{(0)} = 1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$, $\bar{\Psi}_w = -1$	117
5.8	Velocity $\bar{v}^{(0)}$ distribution in the x - y plane for: (a)–(c) variations in $\bar{\kappa}$ with $\bar{\Psi}_w^{(0)} = -1$ and $\bar{B}^{(0)} = 1$: (a) $\bar{\kappa} = 0.1$, (b) $\bar{\kappa} = 0.4$, (c) $\bar{\kappa} = 0.5$. (d)–(f) variations in $\bar{\Psi}_w^{(0)}$ with $\bar{\kappa} = 0.4$ and $\bar{B}^{(0)} = 1$: (d) $\bar{\Psi}_w^{(0)} = -1$, (e) $\bar{\Psi}_w^{(0)} = -1.25$, (f) $\bar{\Psi}_w^{(0)} = -1.5$. (g)–(i) variations in $\bar{B}^{(0)}$ with $\bar{\Psi}_w^{(0)} = -1$ and $\bar{\kappa} = 0.1$: (g) $\bar{B}^{(0)} = 1$, (h) $\bar{B}^{(0)} = 2$, (i) $\bar{B}^{(0)} = 3$	119
5.9	Variation of dimensionless capillary pressure with wetting phase saturation S_w	121
5.10	Figure represent the phase-field distribution of the wetting phase (blue) at different saturation values: (a) $S_w = 0.25593$, (b) $S_w = 0.42836$, (c) $S_w = 0.6123$, and (d) $S_w = 0.80547$. Other constants are $\bar{B}^{(0)} = 1$, $\bar{\nabla}_X \bar{V}^{(0)} = -1$, $\bar{\Psi}_w = -1$, $\bar{\kappa} = 0.1$	121
6.1	(a) 3D reconstructed periodic REV of Whatman filter paper grade 4. (b) 3D domain with transparency showing the internal cellulose fibers, which are treated as solid walls.	128
6.2	Velocity field distribution at multiple (a) z -planes (b) x -planes; Velocity vector distribution (c) in 3D (d) x - y plane for the case when macroscopic magnetic field is 0.628 T.	132

6.3	Velocity field distribution at multiple (a) z-planes (b) x-planes; Velocity vector distribution (c) in 3D (d) x-y plane for the case when macroscopic magnetic field is 0.125 T, $n_0 = 10^{-5}$, $\Psi_w = -50$ mV.	135
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List of Tables

3.1	Mesh convergence study results	55
3.2	Variation of components of electromagneto(EM)-permeability tensor ($K_{em,ij}$) with aspect ratio (α) for porosity = 0.8, $\theta = 0^\circ$ and $B^{(0)} = 1.25T$	58
3.3	Variation of electromagneto permeability tensor components with param- eters $AR(\alpha)$ for different porosity n	60
3.4	Variation of electromagnetic permeability tensor components with rotation angle θ	61
6.1	EM-Permeability values of Whatman Filter Paper Grade 4 under the effect of external electric and magnetic fields.	131
6.2	EM-Permeability values of Whatman Filter Paper Grade 4 under the effect of external electric and magnetic fields.	134

Symbols

a	Semi-axis lengths in x -direction of elliptical solid phase
b	Semi-axis lengths in y -direction of elliptical solid phase
$\mathbf{B}$	Magnetic flux density
$\mathbf{E}$	Electric field
e	Charge on an electron
$\mathbf{F}_B, \mathbf{F}_L, \mathbf{F}_e$	Body force, Lorentz force and electroosmotic force, respectively
$\mathbf{H}$	Magnetic field intensity
$\mathbf{J}$	Current density
k_B	Boltzmann Constant
l	Microscopic length scale
L	Macroscopic length scale
n	Porosity
n_0	Net charge density
nw	Non-wetting phase
p	Pressure
T	Temperature in Kelvin
u, v, w	Velocity along x, y and z directions, respectively
$\mathbf{v}$	Velocity vector
V	Electric potential
X	Macroscopic coordinate system
X_1, Y_1, Z_1	Coordinates in macroscopic scale
x, y, z	Coordinates in microscopic scale
Y	Microscopic coordinate system
z_c	Valency of ion

Greek Letters

α	Aspect ratio of elliptical solid phase
α_n	Fluid phase within the periodic domain
β	Solid phase within the periodic domain
γ	Surface tension of fluid-fluid interface
Γ	Boundary of the periodic domain
Γ_{fs}	Fluid-solid boundary
Γ_{wn}	Fluid-fluid interface
δ	Fluid-fluid interface thickness
ϵ	Electrical permittivity of the electrolyte
ε	Small parameter for asymptotic homogenization
ζ	Zeta potential of the surface
η	Coefficient of cell problem in the potential corrector equation
θ	Orientation angle of solid inclusion
κ	Debye parameter, inverse of Debye layer thickness
λ_D	Debye layer thickness
μ	Dynamic viscosity of fluid
μ^*	Magnetic permeability
ξ_p, ξ_V, ξ_ψ	Coefficients of cell problem in the pressure corrector equation
Ξ_p, Ξ_V, Ξ_ψ	Average pressure corrector coefficients
Π_p, Π_V, Π_ψ	Average potential corrector coefficients
ρ	Density of the fluid
ρ_e	Free charge density
σ	Electrical conductivity of the fluid
ϕ	Phase field variable
ψ	Auxiliary phase field variable
Ψ	Electrical double layer potential
Ω_f, Ω_s	Periodic domain of fluid and solid phases.