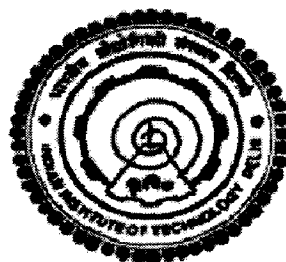


SIGNAL-MATCHED WAVELETS: THEORY AND APPLICATIONS

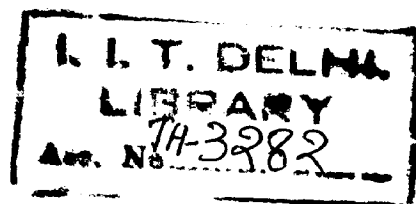
by
Anubha Gupta

Submitted
in fulfillment of the requirements of the degree of Doctor of Philosophy
to the "



Electrical Engineering Department
Indian Institute of Technology, Delhi, India.
December, 2004.

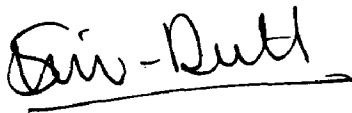
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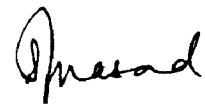
Certificate

This is to certify that the thesis entitled "**Signal-Matched Wavelets: Theory and Applications**" being submitted by Ms. Anubha Gupta to the Department of Electrical Engineering, Indian Institute of Technology, Delhi is the record bonafide work carried out by her. She has worked under our supervision and guidance during the period July 1999 to December 2004. She has fulfilled the requirements for submission of the thesis which has reached the requisite standard.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma.



Prof. ShivDutt Joshi
Thesis supervisor
Electrical Engineering Department
Indian Institute of Technology, Delhi
Hauz Khas, New Delhi-110 016.
India.



Prof. Surendra Prasad
Thesis supervisor
Electrical Engineering Department
Indian Institute of Technology, Delhi
Hauz Khas, New Delhi-110 016.
India.

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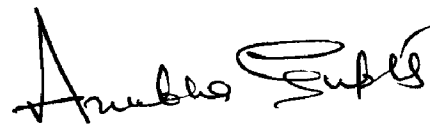
Most importantly, my heartfelt gratitude is for my parents, who were the motivation behind me. Without their blessings, this work could not have been accomplished.

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Anubha Gupta

To my mother

Abstract

This thesis is an attempt to solve the problem of signal-matched wavelets for best signal representation for both 1-D and 2-D signals. The thesis addresses the issues of estimating both, wavelet matched to a deterministic signal and wavelet matched to a signal in statistical sense.

For the deterministic signal case, a new approach based on maximizing projection of the given signal onto successive scaling subspace is proposed for the estimation of wavelet matched to the given signal. Based on this approach, new methods are presented that estimate bi-orthogonal, compactly supported as well as semi-orthogonal, compactly/infinately supported signal-matched wavelets from the given signal. From signal processing perspective, it is important to design a perfect reconstruction filter bank. Therefore, methods are proposed that help in the design of corresponding analysis and synthesis filters such that the resulting 2-band structure is a perfect reconstruction FIR/IIR filter bank. This leads to the estimation of bi-orthogonal as well as semi-orthogonal compactly supported/Infinitely supported wavelet. The proposed method has the flexibility to design wavelet with desired support as well as desired number of vanishing moments.

Further, in all applications, it may not be desirable to estimate wavelet matched to a given signal. Rather, it may be more appropriate to design a wavelet matched to a class of signals. Therefore, it is natural to seek statistically matched wavelet. The thesis proposes a new approach for the estimation of wavelet that is matched to a given stochastic signal in the statistical sense. Since $1/f^\beta$ processes simultaneously exhibit statistical scale invariance and a particular notion of time invariance, wavelet like bases having both scaling and shifting are naturally the best candidate for wavelet based representation. Therefore, assuming the signal to belong to the class of $1/f^\beta$ processes, analysis wavelet filter is

estimated from a given signal using input signal statistics. Next, same methods as proposed above are used to design perfect reconstruction FIR/IIR filter bank such that resulting statistically matched wavelet is both bi-orthogonal as well as semi-orthogonal compactly supported/infininitely supported. While working on this problem, an interesting observation is made on the auto-correlation matrix of discrete-time $1/f^B$ processes. Since, these processes are in general non-stationary; the autocorrelation matrix is a function of time. Here, it is observed that all but one eigenvalues of this autocorrelation matrix are constant with time for sufficiently large value of time index n . The value of only one eigenvalue depends on the time index and it increases as the time index of the autocorrelation matrix increases.

Next, the thesis proposes a new approach to estimate statistically matched uniform M-band wavelet system. The resulting M-band structure is a perfect reconstruction filter bank wherein all the analysis filters are estimated from the given signal using its statistical properties. The corresponding synthesis filter bank is designed using polyphase decomposition matrices of analysis filter bank and consists of IIR filters that are implemented as anti-causal, stable filters. The wavelets associated with this structure are infinitely supported and are statistically matched to the signal.

For 2-D signals, several techniques proposed so far in literature design wavelets independent of the signal under consideration. Therefore, this thesis addresses the issue of estimating matched wavelet for 2-D signals. New methods are proposed for estimation of compactly supported wavelet for the case of separable kernel as well as non-separable kernel from a given image. For the case of non-separable wavelets, the design method is proposed for different sampling lattices. Since the wavelets are estimated from a given image, they are called 2-dimensional matched wavelets. It is further shown that non-separable matched wavelets estimated with the decimation matrix chosen using the

principal axes of the image give much better results as compared to other sampling lattices in the application of compression.

During the course of this work, we arrived at a theorem that proposes a relationship between scaling filter and wavelet filter for 2-channel non-separable orthogonal wavelet system in $\mathbb{R}^2$ for any decimation matrix. An explicit relation between these filters can be derived in time domain using the proposed relation in frequency domain.

Table of Contents

Certificate.....	i
Acknowledgement.....	ii
Abstract.....	iii
List of Figures.....	x
List of Tables.....	xii
Chapter 1: Introduction.....	1
1.1 Motivation	1
1.2 Literature Survey.....	3
1.3 Problem Definition and Scope of the Thesis	6
1.4 Organization of the thesis.....	9
Chapter 2: Preliminaries.....	12
2.1 Brief Overview of Multiresolution Signal Approximation & DWT.....	12
2.1.1 Introduction.....	12
2.1.2 Orthogonal Multiresolution Signal Approximation.....	12
2.1.3 Discrete Wavelet Transform.....	14
2.1.4 Discrete Wavelet Transform for 2-D Signals.....	15
2.2 Review of Multirate Filter Bank Basics.....	18
2.2.1 Introduction.....	18
2.2.2 M-Channel Uniformly Decimated Filter Bank.....	18
2.2.3 Relation between Wavelets and Filter Bank Theory.....	20
2.3 A Brief Introduction to the Theory of Self Similar Random Processes.....	24
2.3.1 Introduction.....	24
2.3.2 Self Similar Random Processes.....	24
2.3.3 Fractional Brownian motion.....	25

2.3.4	The Generalized m^{th} Order Fractional Brownian Motion and Fractional Gaussian Noise.....	27
2.3.5	Estimation of Hurst parameter.....	28
2.4	Conclusions.....	28
Chapter 3: A Novel Approach to Estimation of Matched Wavelet from a given		
	Signal.....	29
3.1	Estimation of Matched Analysis Wavelet Filter.....	30
3.2	Bi-orthogonal Compactly Supported Matched Wavelet.....	34
3.2.1	Design of FIR Perfect Reconstruction Biorthogonal Filter Bank.....	34
3.2.2	Design of Bi-orthogonal wavelet with desired support and vanishing Moments.....	34
3.2.3	Algorithm-1.....	35
3.2.4	Simulation Results.....	36
3.3	Semi-orthogonal Matched Wavelet.....	41
3.3.1	Design of Semi-Orthogonal matched wavelet.....	42
3.3.2	Algorithm-2.....	43
3.3.3	Simulation Results.....	44
3.4	Wavelets and binary tree structured filter banks.....	46
3.5	Simulation Results in the context of Signal Compression.....	48
3.6	Conclusions.....	49
Chapter 4: A New Approach for the Estimation of Statistically Matched Wavelet....		
4.1	Compactly Supported Statistically Matched Bi-orthogonal 2-Band Wavelet System.....	52
4.1.1	Estimation of Matched Analysis Wavelet Filter using signal statistics.....	52
4.1.2	Design of FIR Perfect Reconstruction Biorthogonal Filter Bank.....	55

4.1.3	Design of Bi-orthogonal wavelet with desired support and desired number of vanishing moments.....	56
4.1.4	Algorithm-1	56
4.1.5	Simulation Results.....	57
4.2	Statistically Matched Semi-orthogonal 2-Band Wavelet System.....	61
4.2.1	Estimation of Matched Analysis Wavelet Filter.....	61
4.2.2	Design of Semi-Orthogonal matched wavelet	61
4.2.3	Algorithm-2.....	63
4.2.4	Simulation Results.....	63
4.3	Estimation of Statistically Matched Wavelet for Uniform M-band Wavelet System.....	67
4.3.1	Estimation of Matched Analysis Wavelet Filter.....	67
4.3.2	Design of Matched Bandpass Filters and Lowpass Filter of Analysis Filter Bank.....	70
4.3.3	Design of M-band Perfect Reconstruction Biorthogonal Filter Bank with IIR synthesis filters.....	72
4.3.4	Algorithm-3.....	73
4.3.5	Simulation Results.....	75
4.4	Simulation Results in the context of signal Compression	78
4.5	Conclusions.....	81
Chapter 5: Proposed Approach to Estimation of Wavelets Matched to 2-D Signals.....		
5.1	Estimation of Wavelet with Non-Separable Kernel for 2-D signals.....	84
5.1.1	Estimation of Matched Analysis Wavelet Filter.....	84
5.1.2	Design of FIR Perfect Reconstruction Biorthogonal Filter Bank.....	88

5.1.3	Design of Compactly Supported Wavelet with different Sampling Lattices/ Shapes.....	89
5.1.4	Algorithm-1.....	90
5.1.5	Simulation Results.....	91
5.2	Estimation of Wavelet with Separable Kernel for 2-D signals.....	93
5.2.1	Estimation of Matched Wavelet.....	93
5.2.2	Algorithm-2.....	93
5.3	Simulation Results in the Context of Image Compression.....	94
5.4	Conclusions.....	98
Chapter 6: Additional Results.....		99
6.1	Relation between Scaling Filter and Wavelet Filter In $\Re^2$.....	99
6.1.1	Introduction.....	99
6.1.2	Proposed Theorem.....	100
6.1.3	Design Examples.....	104
6.1.4	Conclusions.....	107
6.2	Structure of Autocorrelation Matrix of discrete-time $1/f^\beta$ process	107
6.2.1	Introduction.....	107
6.2.2	Proposed Theorem.....	107
6.2.3	Conclusions.....	110
Chapter 7: Conclusions.....		111
7.1	Summary of results.....	111
7.2	Scope for Future Work.....	113
References.....		115
List of publications.....		121